How (not) to parameterize two-body amplitude

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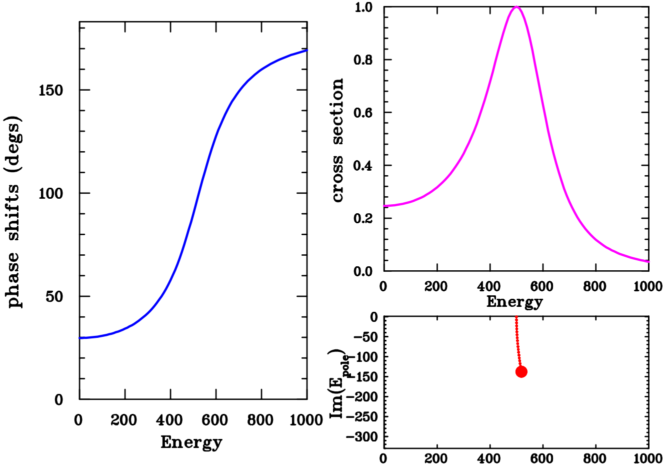
Institute of Nuclear Physics PAS, Kraków

GSI I 2019

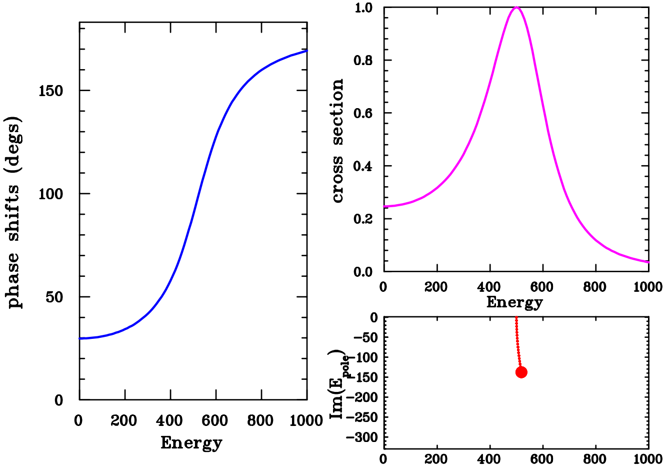
Resonances



phase shifts (degs)
0
50
100
150
0
200
400
600
800
1000
Energy
cross section
0.0
0.2
0.4
0.6
0.8
1.0
0
200
400
600
800
1000
Energy
Im(E_pole)
0
−50
−100
−150
−200
−250
−300
0
200
400
600
800
1000

Resonances: Breit-Wigner (BW) approximation:

$$Ampl = \frac{C}{M_{BW} - E - i\Gamma_{BW}/2}, \quad \delta = ArcTan\left(\frac{\Gamma/2}{M_{BW} - E}\right), \quad \sigma = \frac{C^2}{(M_{BW} - E)^2 + \Gamma^2/4}$$



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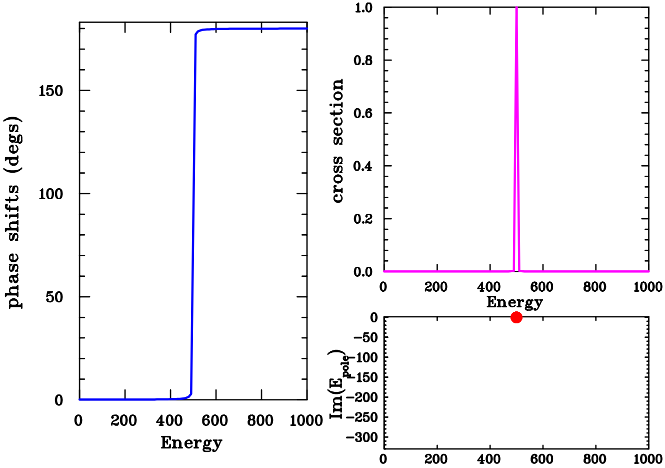

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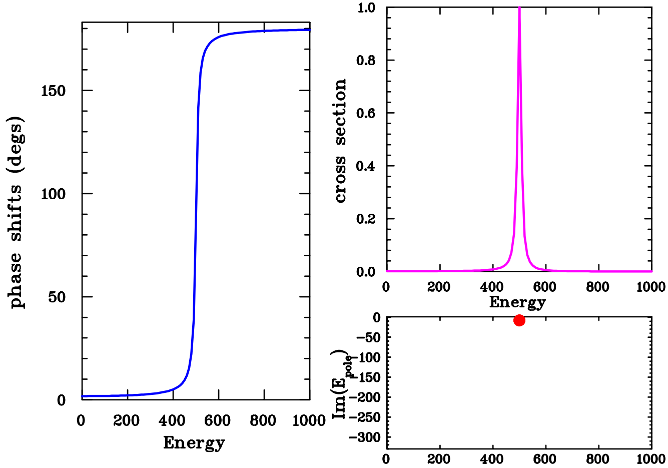

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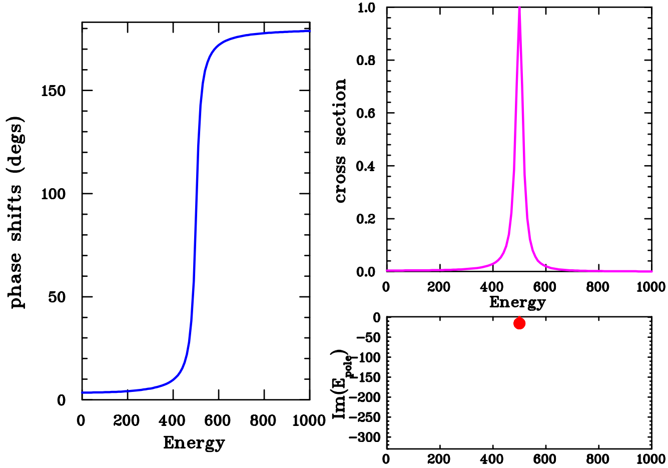

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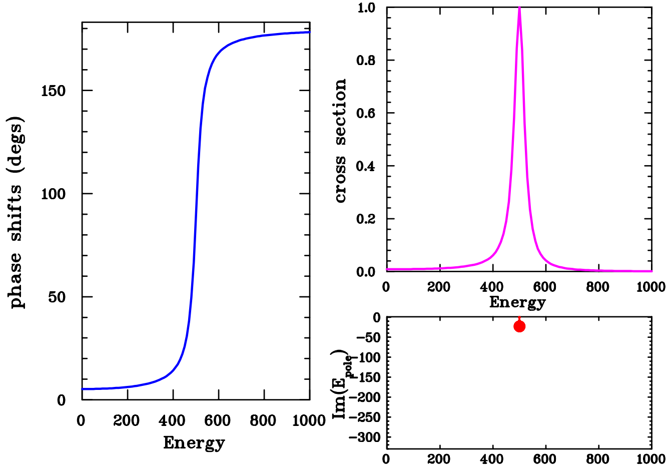

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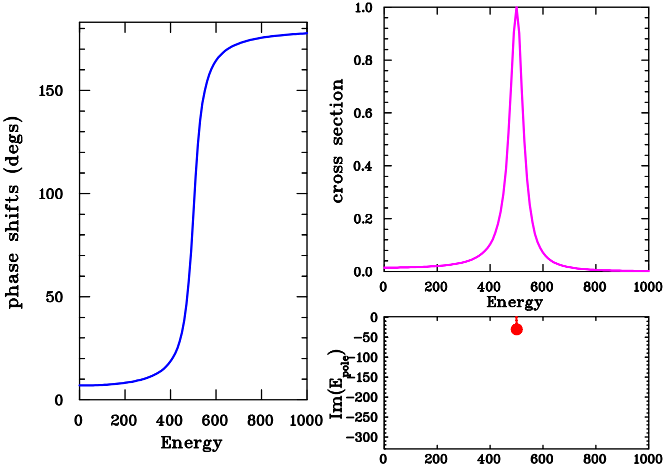

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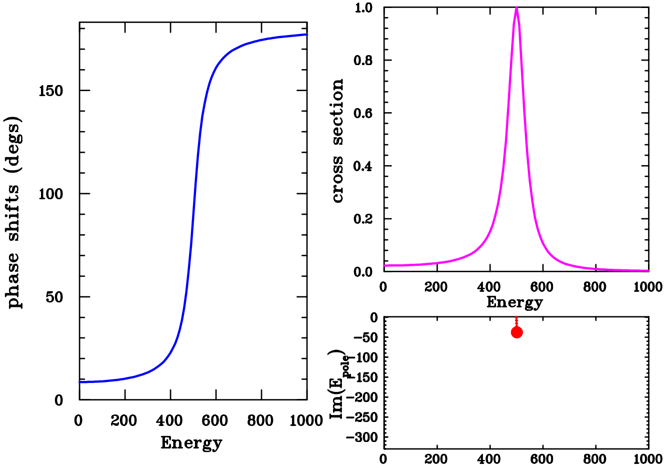

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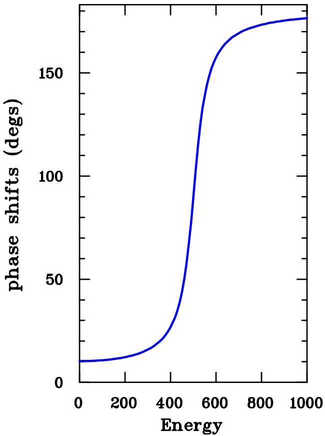

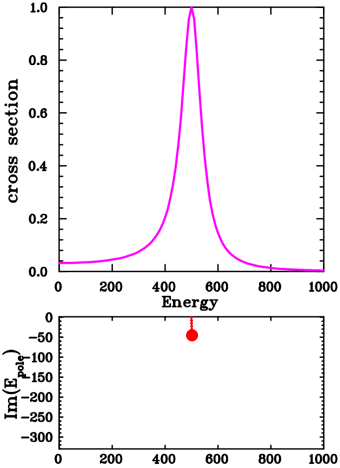

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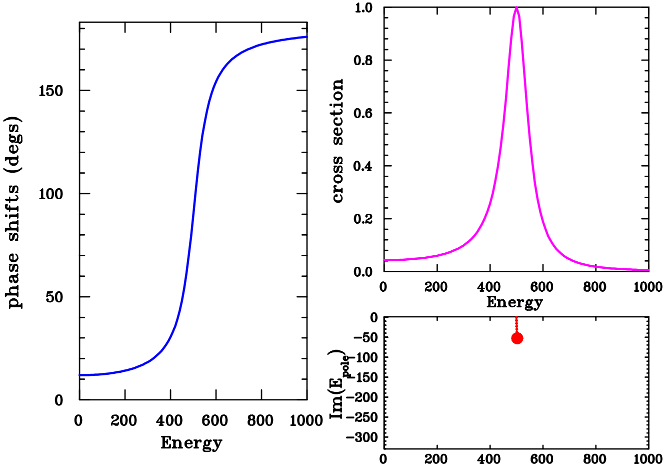

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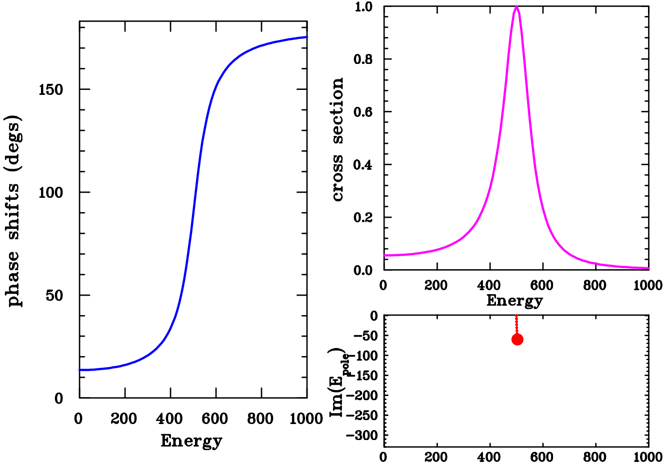

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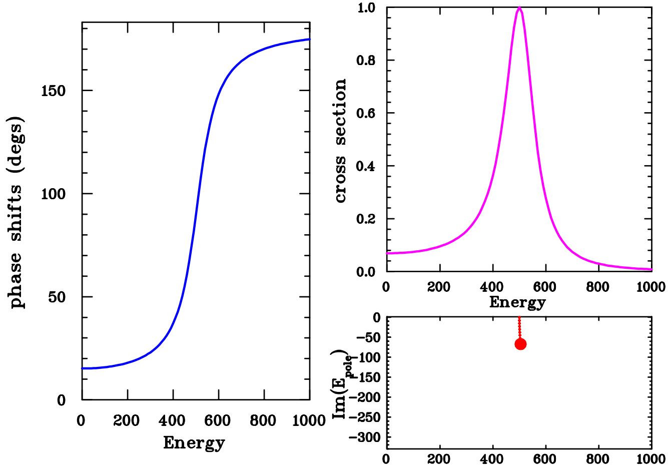

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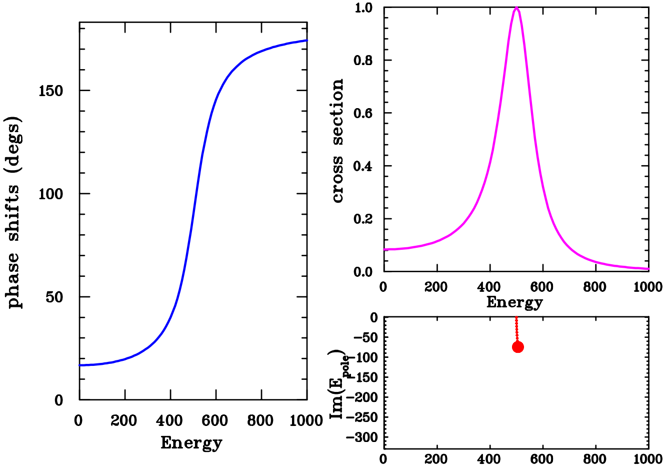

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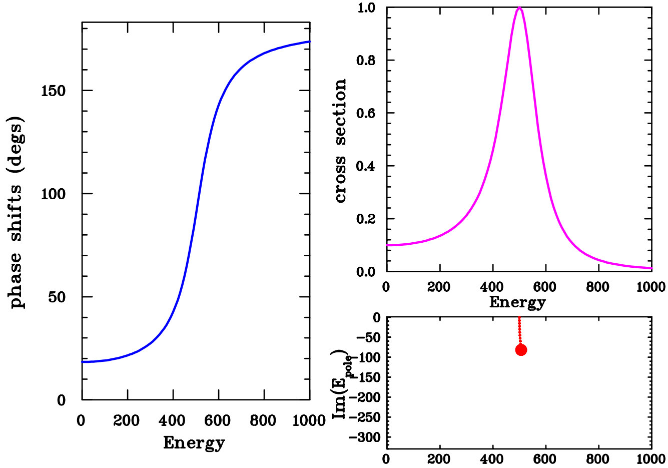

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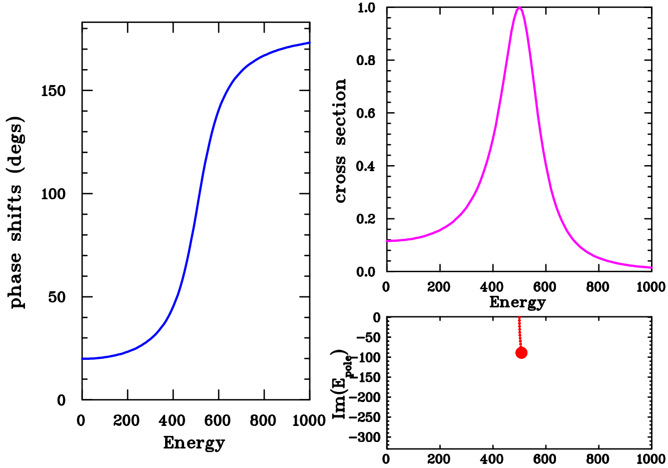

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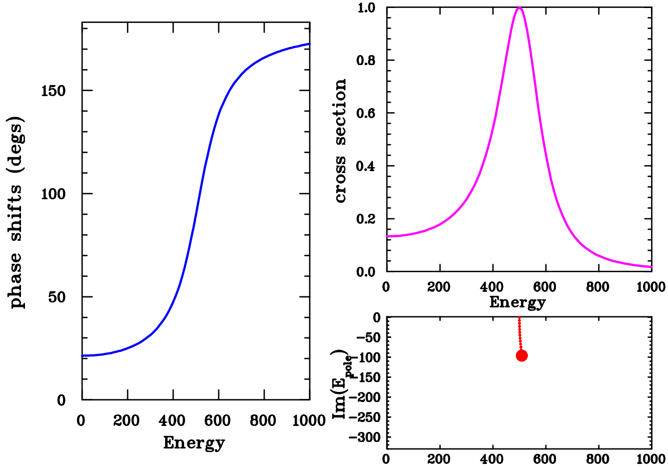

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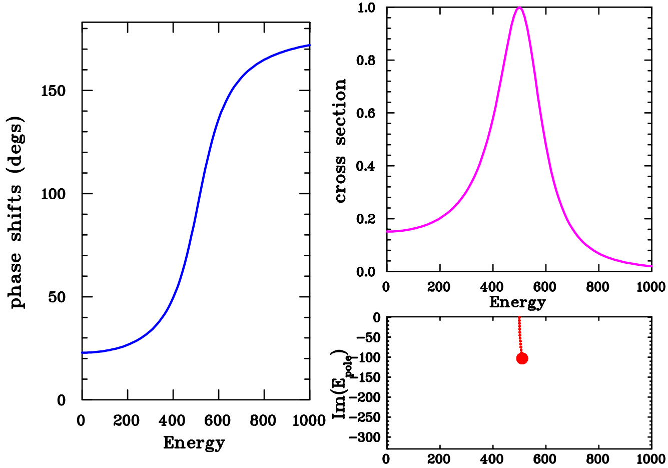

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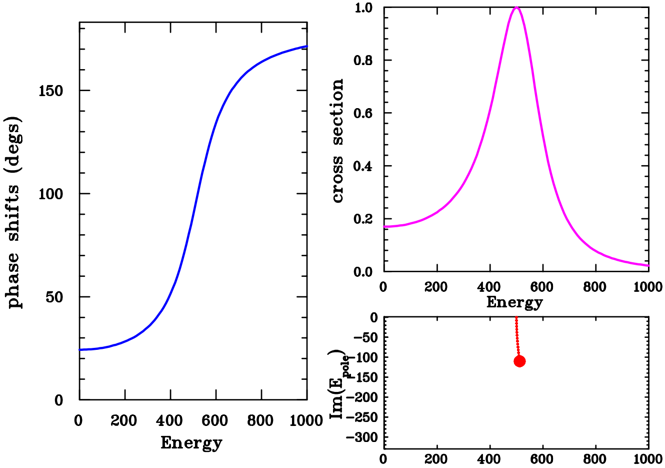

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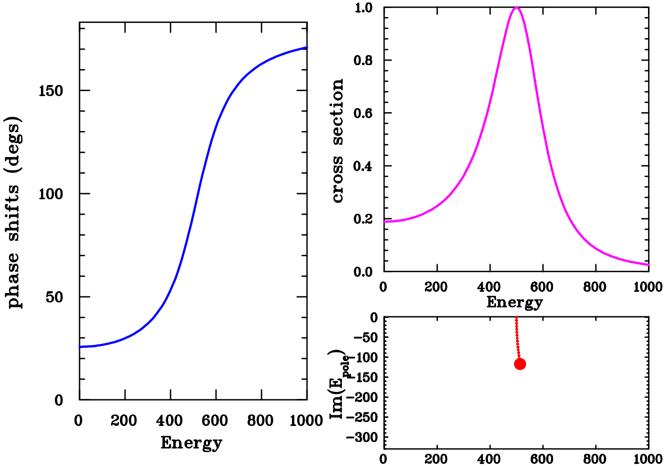

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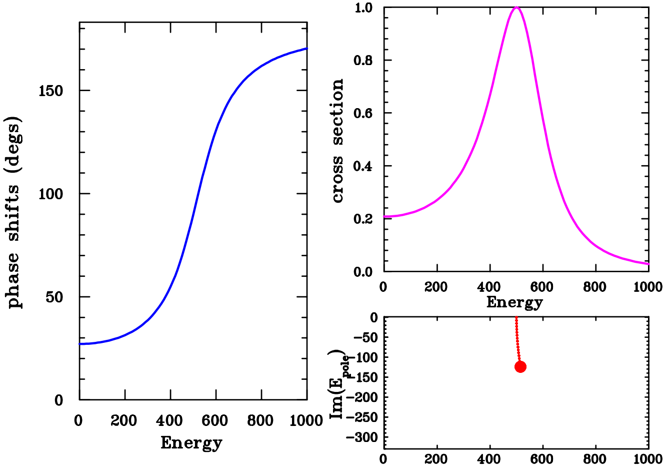

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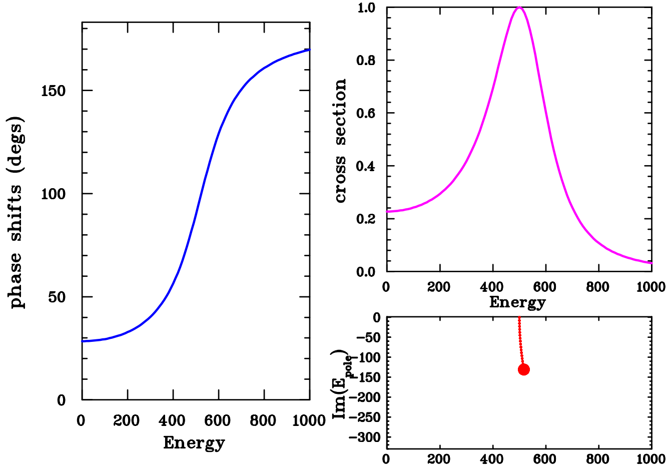

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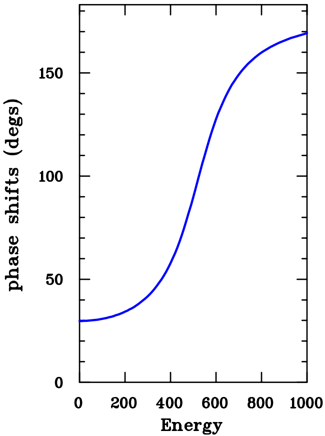

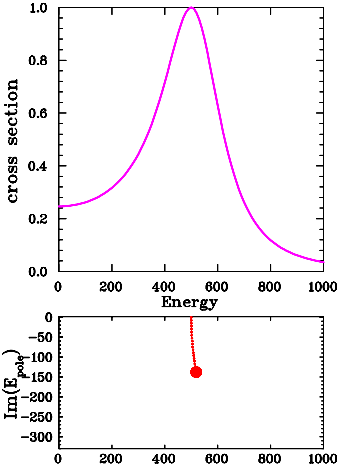

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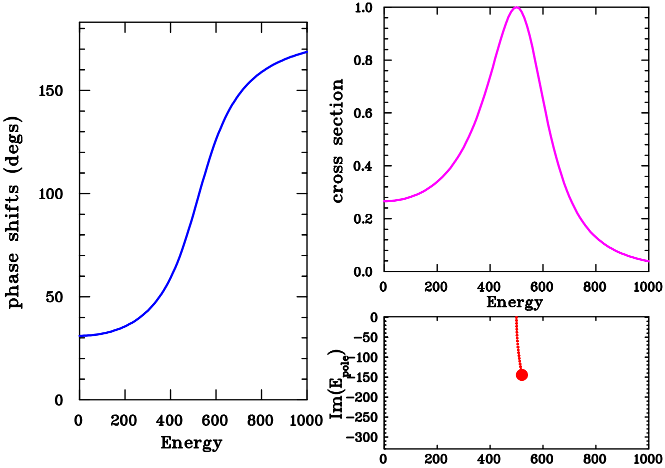

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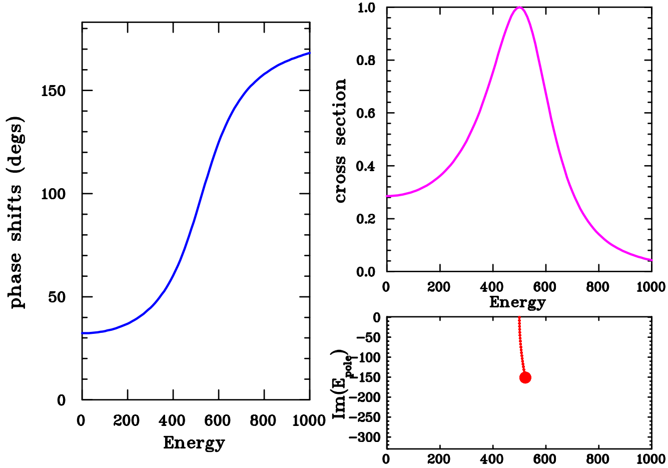

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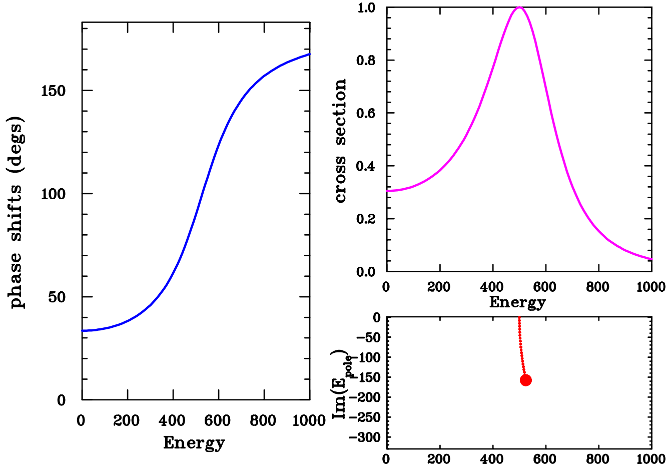

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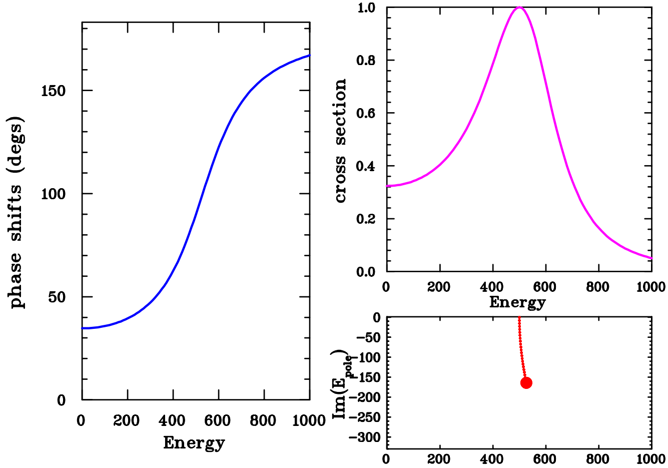

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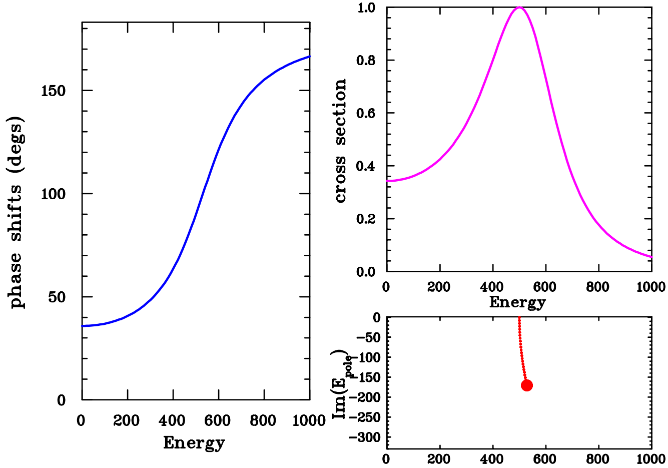

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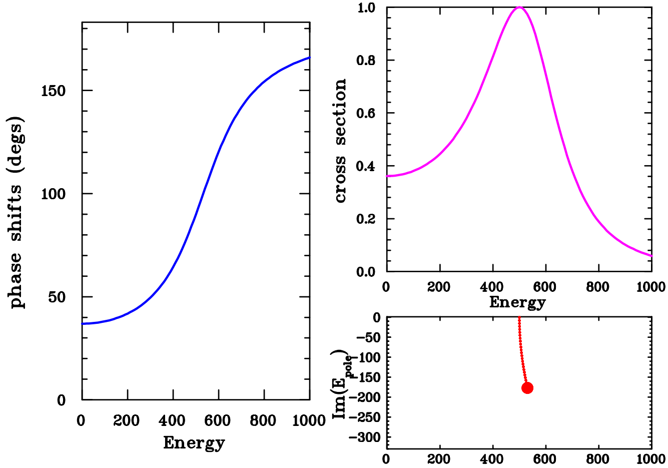

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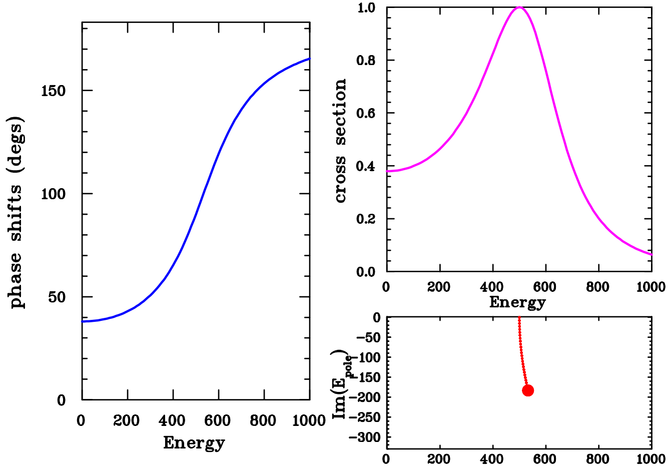

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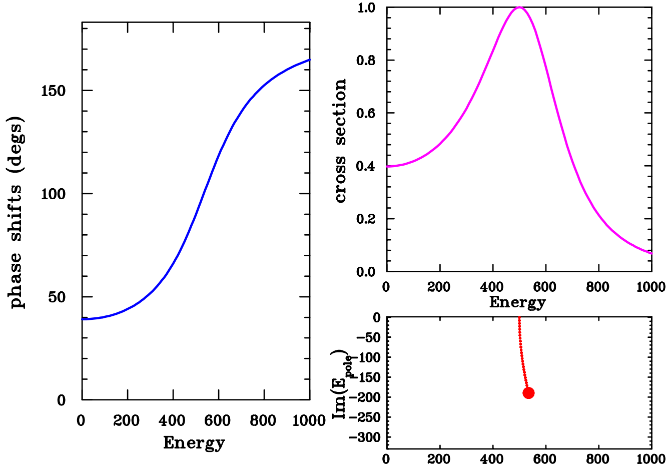

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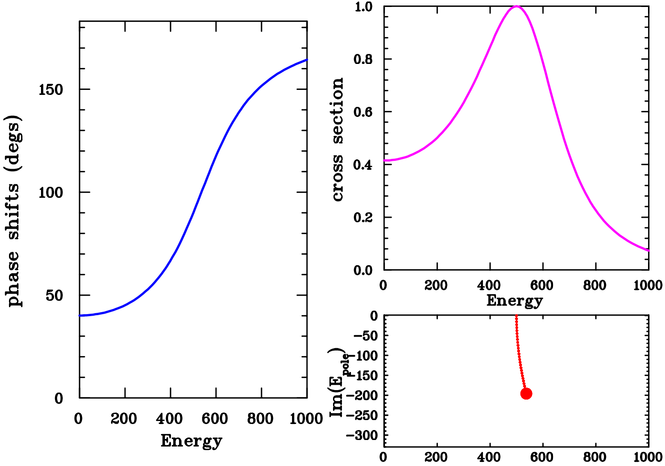

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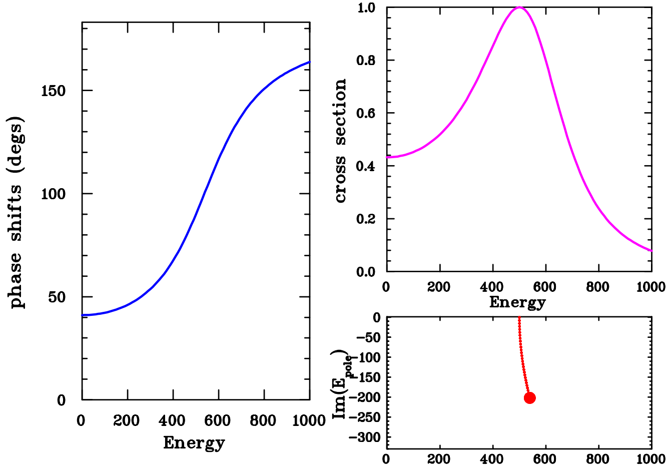

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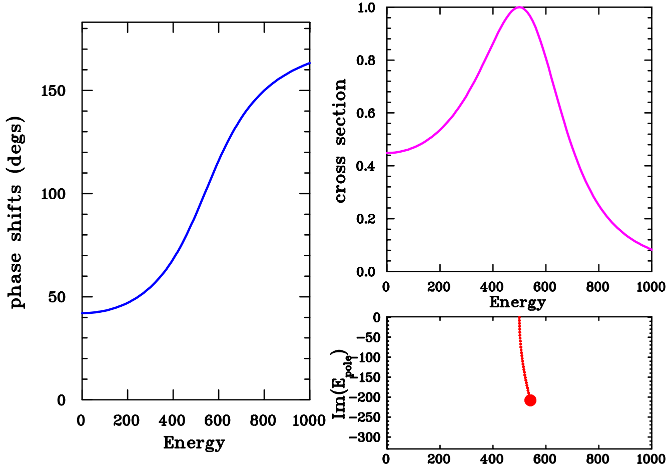

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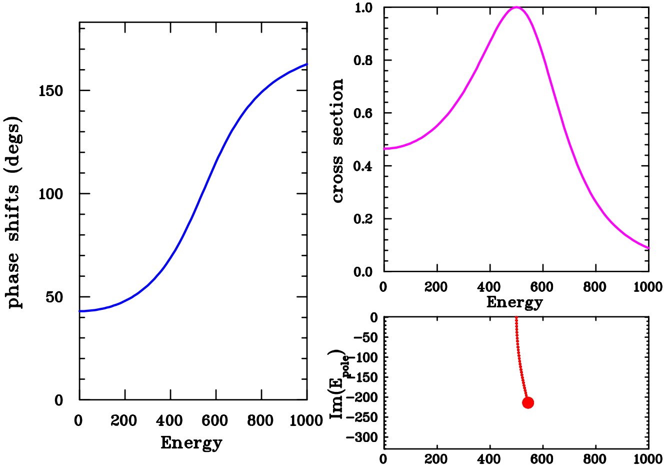

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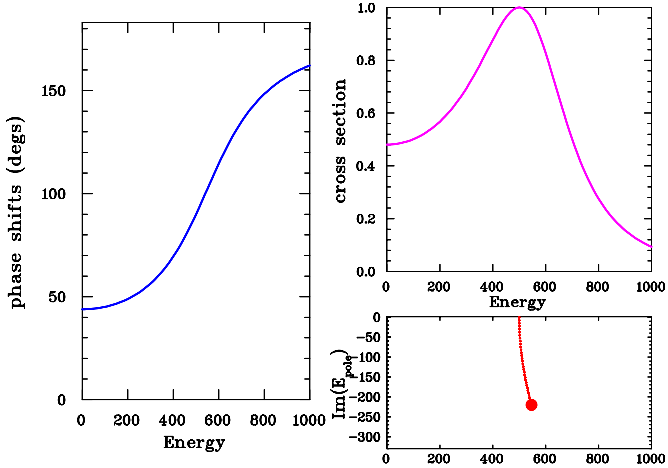

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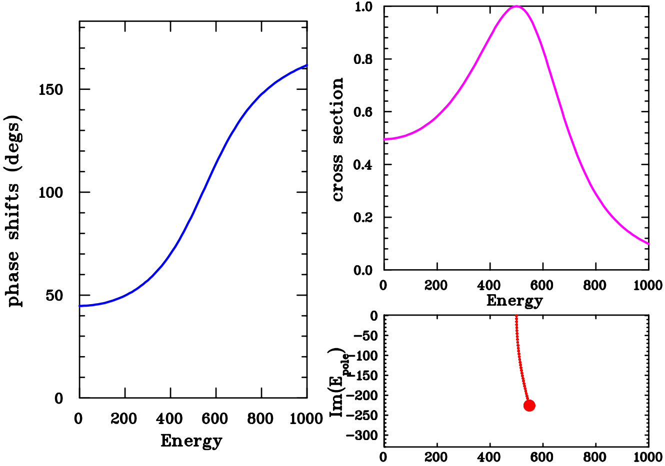

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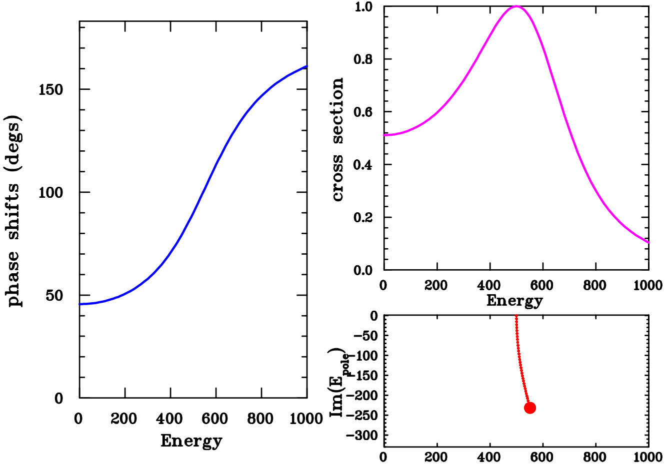

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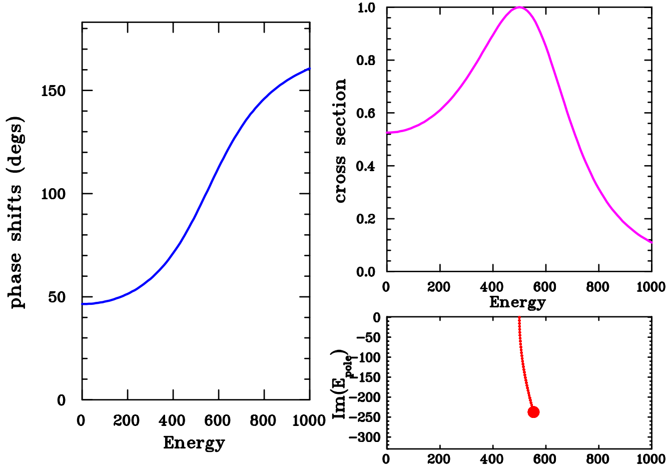

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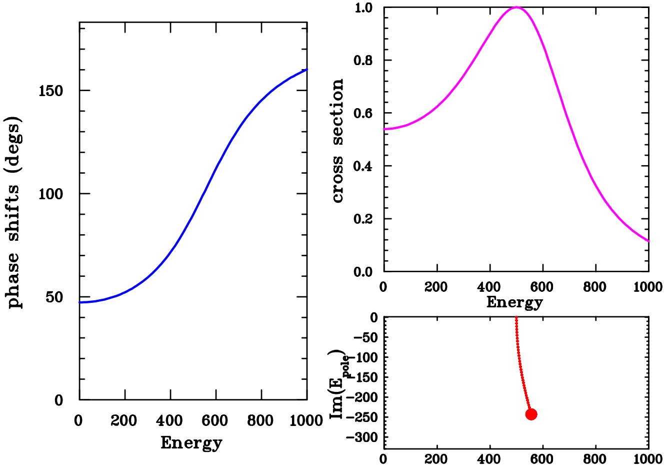

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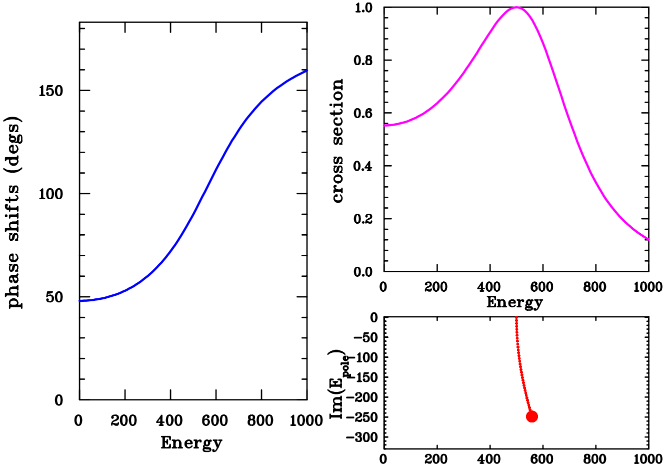

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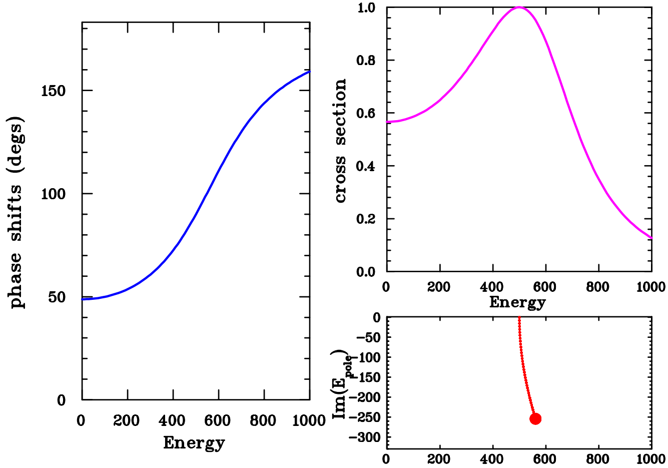

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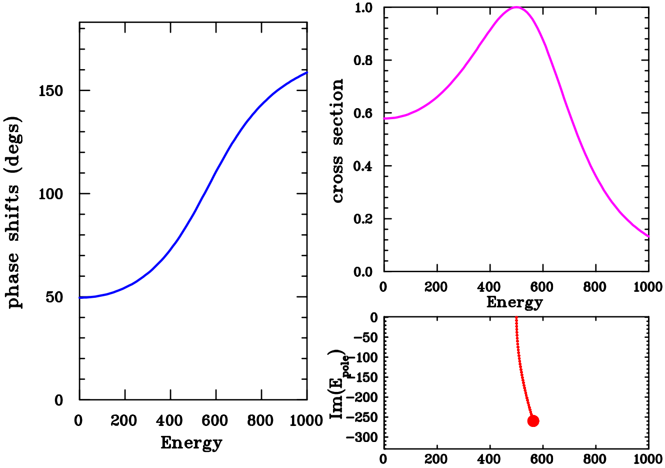

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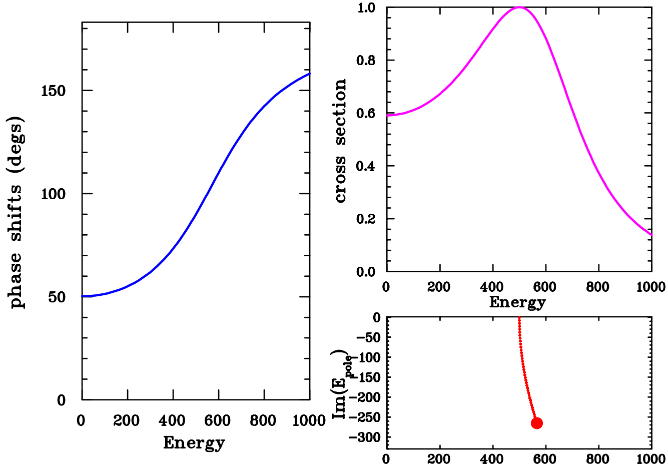

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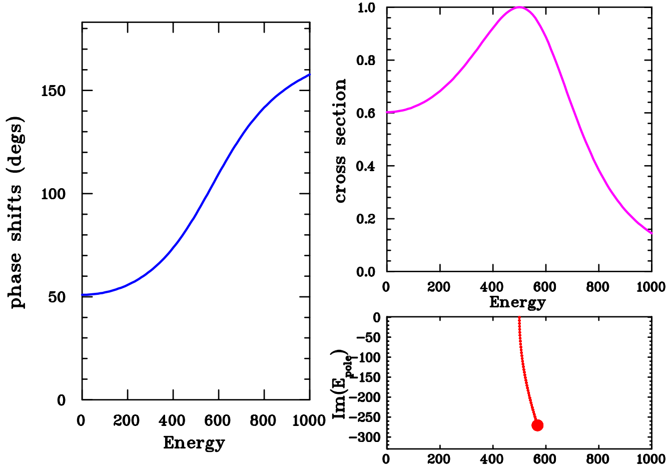

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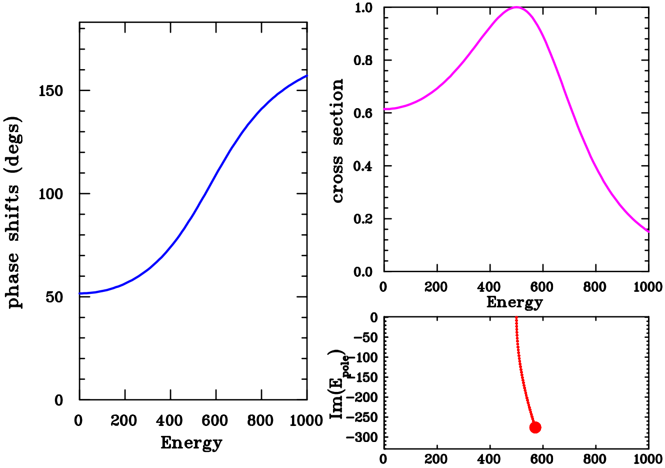

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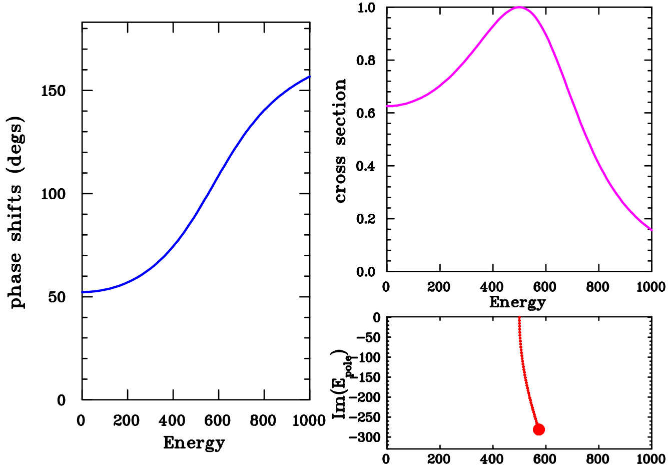

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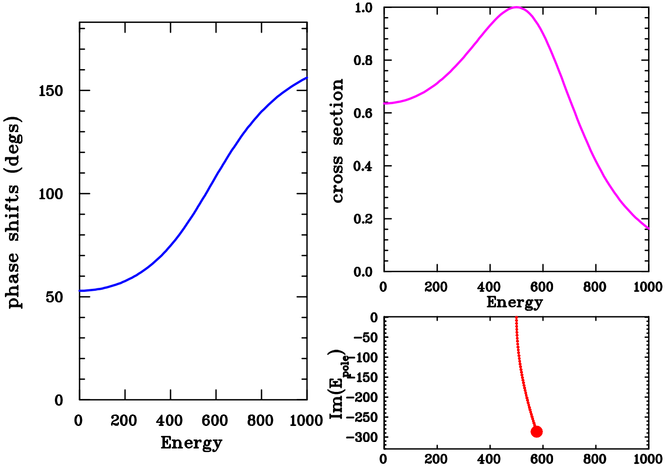

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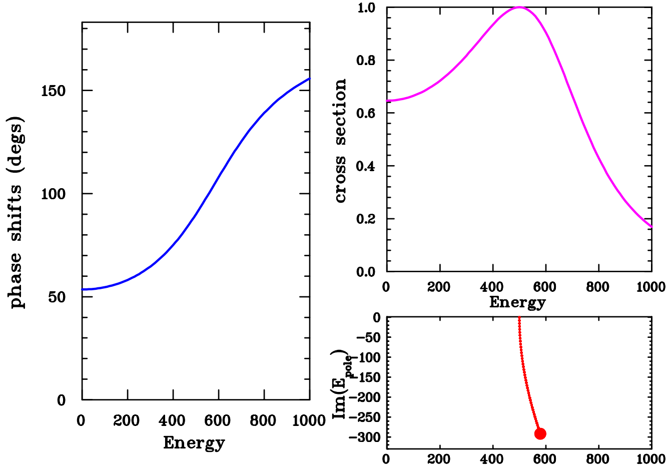

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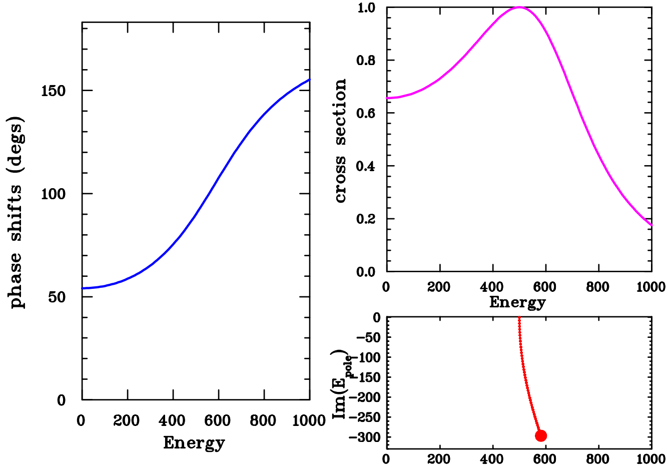
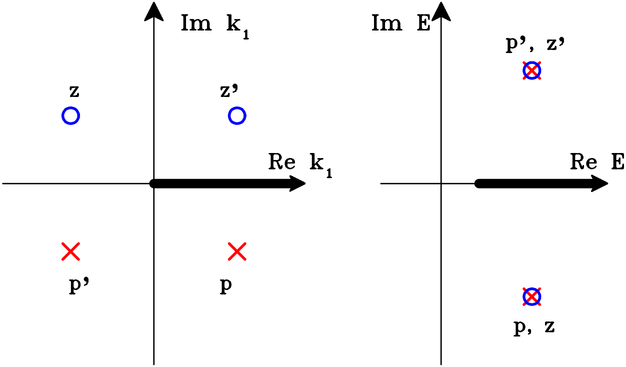


Complex momentum and energy space frame

$$E = 2\sqrt{(\pm k)^2 + m^2}$$



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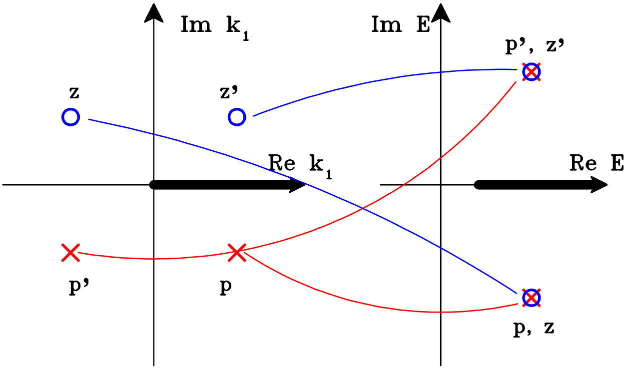

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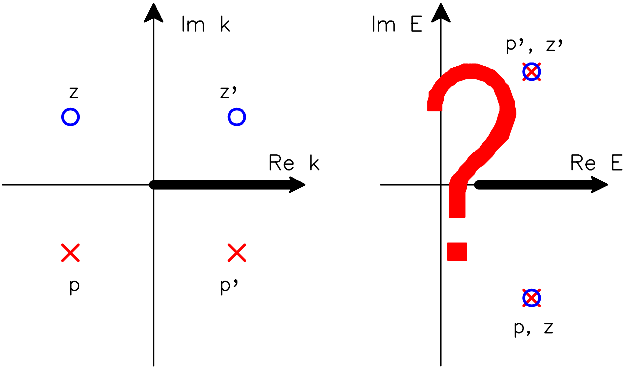

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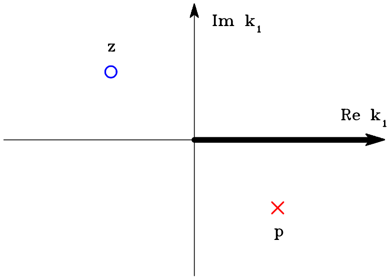

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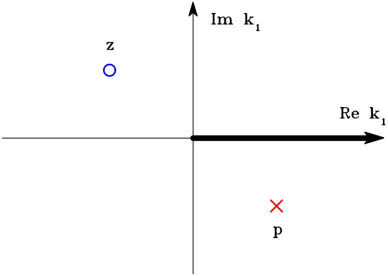

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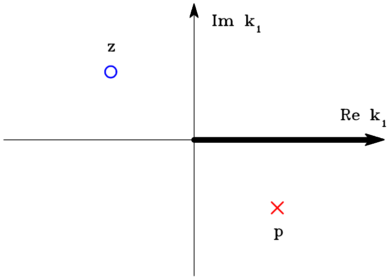

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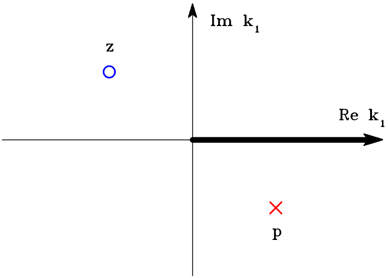

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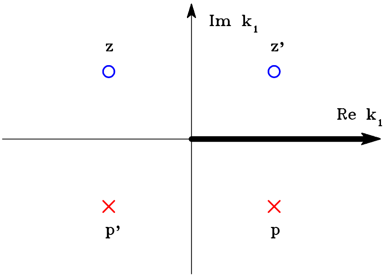

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- then $|S(k)| = 1$
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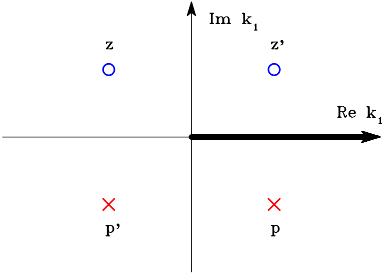

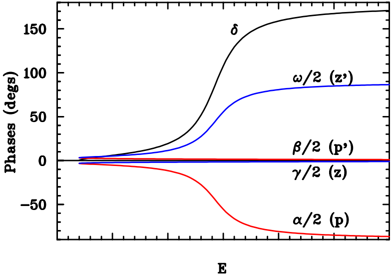

Breit Wigner approximation

- $BW(E) = \frac{\Gamma/2k}{M_{BW} - E - i\Gamma/2}$
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Pole and mass of a resonance

- Let's imagine good fit of an amplitude to the data —> mass M_{BW} at $\delta = 90^o$
  - Let's fit amplitude $A_{S_{notU}}$ to the same data. $A_{S_{notU}}$ has a single pole at $k_j = a - ib$ then $\delta = ArcTan(\frac{-b}{k-a}) + ArcTan(\frac{b}{-k-a})$ and $M_{BW} \neq 2\sqrt{a^2 + m^2}$, additionally $|S| \neq 1$
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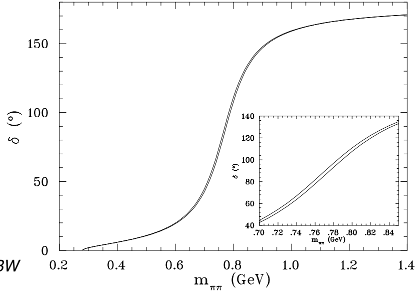
Let's check it for $\rho(770)$:
$M_{BW} = 775.26 \pm 0.25$ MeV (PDG'2016),
$\Gamma = 149.1 \pm 0.8$ MeV (PDG'2016),
$2\sqrt{a^2 + m^2} < M_{BW}$ by ≈ 9 MeV !!!

Left (upper) line:
A_{BW} fitted to the data

Right (lower) line:
A_S fitted to the data with $2\sqrt{a^2 + m^2} = M_{BW}$



First three remarks

(one channel and one resonance)

1. Breit Wigner approximation is unitary

2. Breit Wigner approximation works well only for single resonances and not far from their maximum

3. Positions of poles and Breit Wigner masses are not the same!

bigger $\Gamma \Rightarrow$ bigger difference between $Re(E_{pole})$ and M_{BW}

More resonances (but still one channel)

Adding resonances (for simplicity two resonances, both with $S = e^{2i\delta}$):

- Isobar model: adding amplitudes (even unitary ones) violates unitarity:
  $T_{1,2} = T_1 + T_2 = \frac{S_1 - 1}{2ik} + \frac{S_1 - 1}{2ik} \to S_1 + S_2 = e^{2i\delta_1} + e^{2i\delta_2}$
  of course $|S_1 + S_2| \neq 1$,
- Product of S matrices: $|S_1 S_2| = 1$ in elastic case and $|S_1 S_2| < 1$ in inelastic case ($S = \eta e^{2i\delta}$)
  For example $S_{1,2} = \frac{(-k-k_1)(-k+k_1^*)(-k-k_2)(-k+k_2^*)}{(k-k_1)(k+k_1^*)(k-k_2)(k+k_2^*)}$
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- Sum of K matrices: $S = 1 + 2iT = (1 + iK)/(1 - iK)$ does noit violate unitarity, for example $T_{1,2} = \frac{1}{k} \frac{K_1 + K_2}{1 - iK_1 - iK_2}$

More channels: $k_2 = \pm\sqrt{k_1^2 + m_1^2 - m_2^2}$

(Im(k_1), Im(k_2)): (+,+), (-,+) ....

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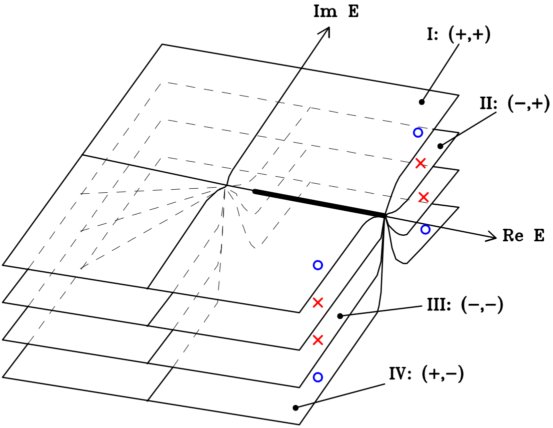

Multiplication and displacement of S matrix poles

- Multiplication:

  1 pole $\longrightarrow 2^{n-1}$ poles due to $(\pm k)^2$ ambiguity and 2^n Riemann sheets

- Displacement:

  $S_{11} = \frac{D_1(-k_1)D_2(k_2)}{D_1(k_1)D_2(k_2)}$ in decoupled case

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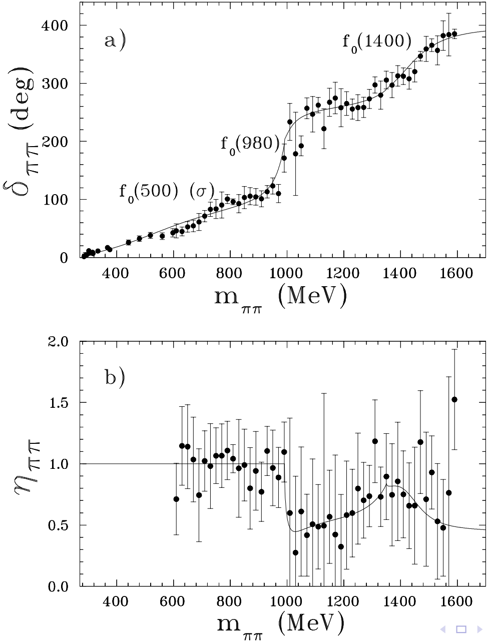
$$S = \begin{pmatrix} \eta e^{2i\delta_1} & i\sqrt{1-\eta^2} e^{i(\delta_1+\delta_2)} \\ i\sqrt{1-\eta^2} e^{i(\delta_1+\delta_2)} & \eta e^{2i\delta_2} \end{pmatrix} = \begin{pmatrix} \frac{D(-k_1,k_2)}{D(k_1,k_2)} & S_{12} \\ S_{21} & \frac{D(k_1,-k_2)}{D(k_1,k_2)} \end{pmatrix}$$

where $S_{12}^2 = S_{21}^2 = S_{11}S_{22} - \frac{D(-k_1,-k_2)}{D(k_1,k_2)}$

Example for two channels: $JI = S0$ wave



Example for two channels: $JI = S0$ wave

Pole	ReE_{pole} MeV	ImE_{pole} MeV	R. sheet
1	639.6	-323.9	$(-,-)$: *III*
1'	511.4	-230.6	$(-,+)$: *II*
2	982.0	-36.9	$(-,+)$: *II*
2'	432.4	-8.4	$(-,-)$: *III*
3	1431.7	-79.3	$(-,-)$: *III*
3'	1394.9	-120.6	$(-,+)$: *II*

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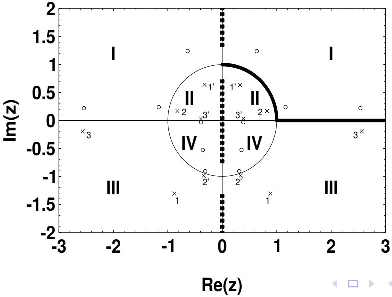
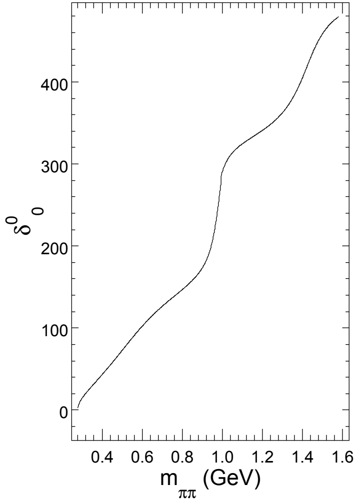
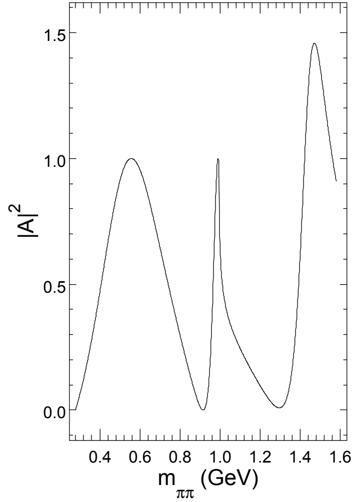
$$z = \frac{k_1 + k_2}{\sqrt{m_K^2 - m_\pi^2}}$$

Rysunek 16: Położenie biegunów (krzyże) i zer (kółka) elementu macierzowego S_{11} macierzy rozpraszania dla dopasowania do zestawu D_{CKM} A. Gruba linia ciągła oznacza obszar fizyczny rozpraszania w kanałach sprzężonych $\pi\pi$ i $K\overline{K}$. Grubą linią przerywaną przedstawione jest położenie cięć funkcji Josta. Cienką linią zaznaczony jest okrąg $|z| = 1$. Numeracja poszczególnych płatów i biegunów została wyjaśniona w tekście.



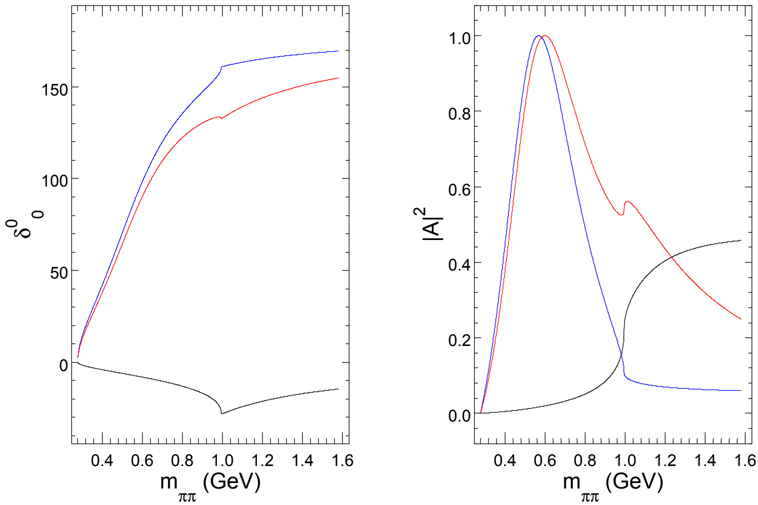
For two channels: ALL POLES 1, 1', 2, 2', 3, 3'





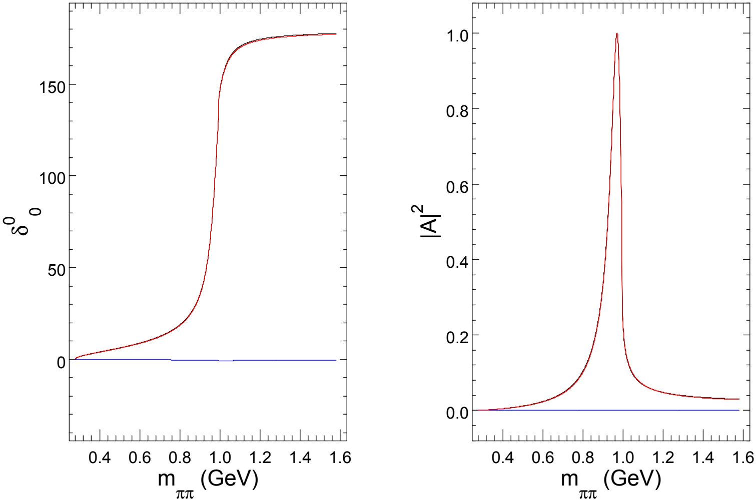
For two channels" TWO POLES 1 and 1' ($f_0(500)$)

1: 639.6-i*323.9   1': 511.4-i*230.6   both 1 and 1'



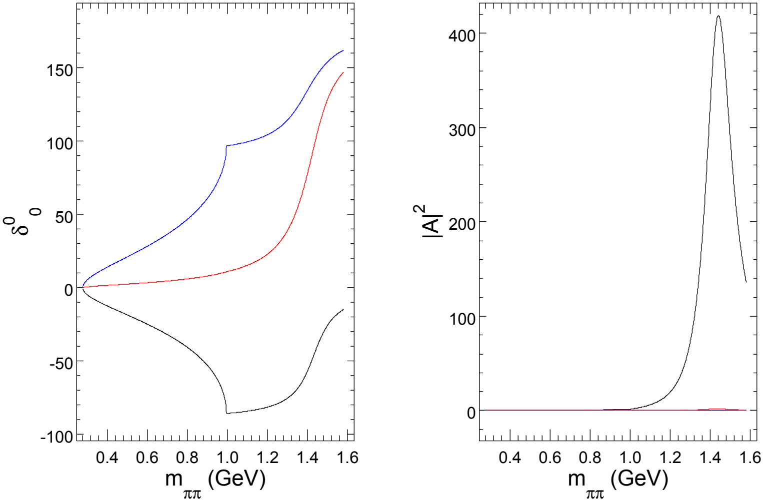
For two channels" TWO POLES 2 and 2' ($f_0(980)$)

2: 982.0-i*36.9 2': 432.4-i*8.4 both 2 and 2'



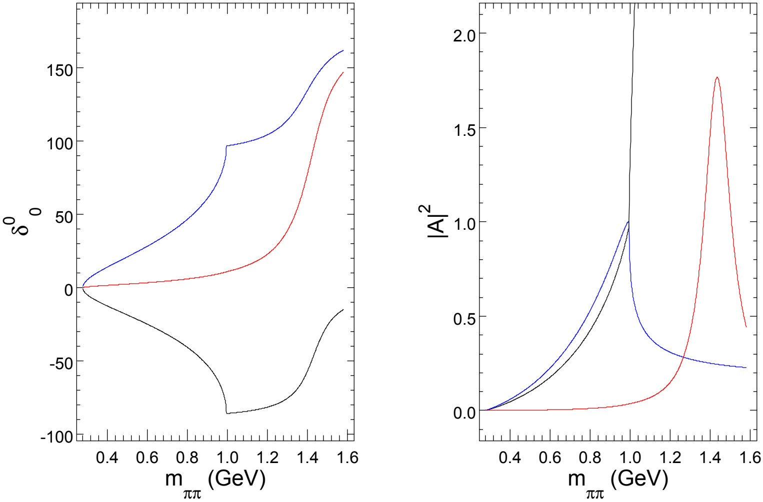
For two channels" TWO POLES 3 and 3' ($f_0(1400)$)

3: 1431.7-i*79.3 3': 1394.9-i*120.6 both 3 and 3'



For two channels" TWO POLES 3 and 3' ($f_0(1400)$)

3: 1431.7-i*79.3   3': 1394.9-i*120.6   both 3 and 3'



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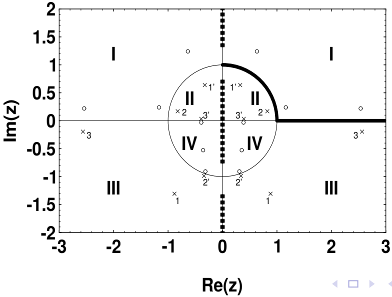


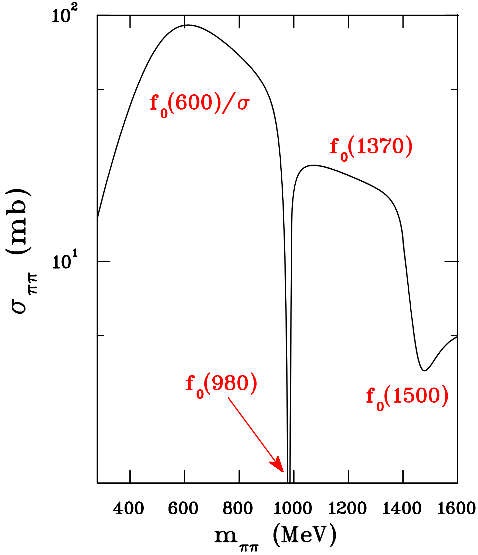
More than 2 channels

- more poles and more Riemann sheets (2^n)
- no similar "z" variable

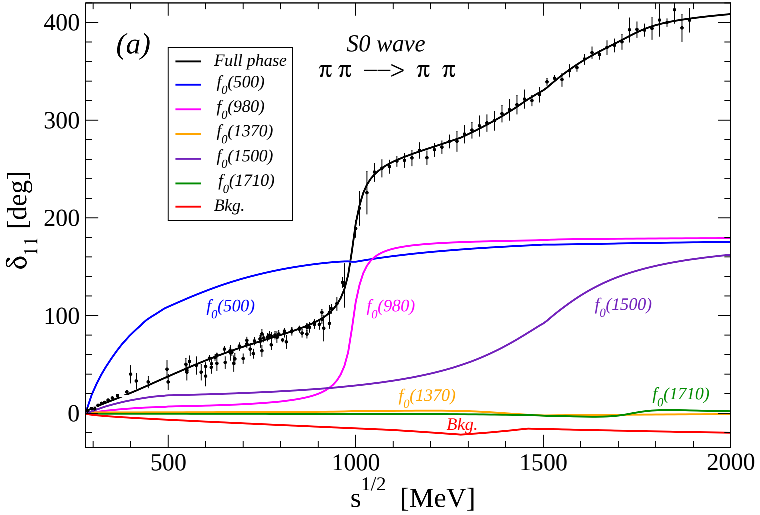
2^n Riemann sheets for n channels

channel	$C = 0$ ReE	$C = 0$ ImE	$C = 1$ ReE	$C = 1$ ImE	sign Imk_π, Imk_K, Imk_3	sheet
$\pi\pi$	658	-607	564	-279	$-,-,-$	VI
			518	-261	$-,+,+$	II
			211	0	$-,+,-$	VII
			532	-315	$-,-,+$	III
			235	0	$+,+,-$	VIII
$\pi\pi$	1346	-275	1405	-74	$-,-,-$	VI
			1445	-116	$-,+,+$	II
			1424	-94	$-,+,-$	VII
			1456	-47	$-,-,+$	III
$K\bar{K}$	881	-498	170	0	$+,-,-$	V
			159	0	$-,-,-$	VI
			418	-10	$-,-,+$	III
			1038	-204	$-,+,-$	VII
			988	-31	$-,+,+$	II
$\sigma\sigma$	118	-2227	4741	-4688	$-,-,-$	VI
			3687	-2875	$-,+,-$	VII
			3626	-3456	$+,-,-$	V
			3533	-579	$+,+,-$	VIII

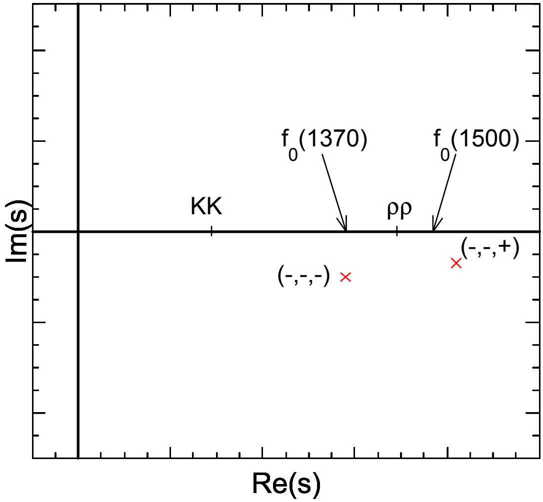
Puzzling $(J I)$ $S0$ wave $\pi\pi$ cross section



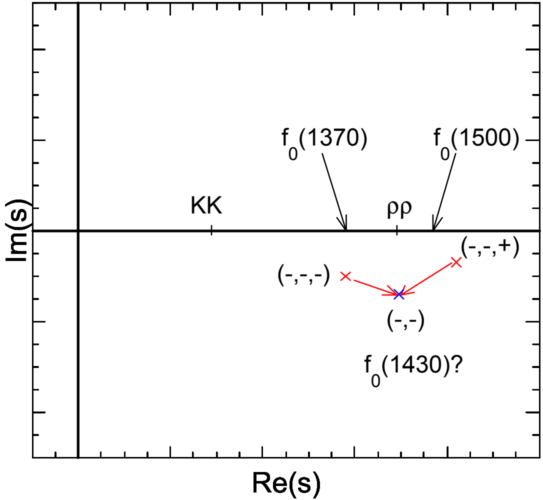
phase shifts of components in the *S*0 wave



$f_0(1370)$ and $f_0(1500)$: positions of poles, $C = 1$



$f_0(1370)$ and $f_0(1500)$: positions of poles, $C = 0$



2^n Riemann sheets for n channels

channel	$C=0$		$C=1$		sign	sheet
	ReE	ImE	ReE	ImE	Imk_π, Imk_K, Imk_3	
$\pi\pi$	658	-607	564	-279	$-,-,-$	VI
			518	-261	$-,+,+$	II
			211	0	$-,+,-$	VII
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			235	0	$+,+,-$	VIII
$\pi\pi$	1346	-275	1405	-74	$-,-,-$	VI
			1445	-116	$-,+,+$	II
			1424	-94	$-,+,-$	VII
			1456	-47	$-,-,+$	III
$K\bar{K}$	881	-498	170	0	$+,-,-$	V
			159	0	$-,-,-$	VI
			418	-10	$-,-,+$	III
			1038	-204	$-,+,-$	VII
			988	-31	$-,+,+$	II
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			3687	-2875	$-,+,-$	VII
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			518	-261	$-,+,+$	II	$\leftarrow f_0(500)$
			211	0	$-,+,-$	VII	
			532	-315	$-,-,+$	III	
			235	0	$+,+,-$	VIII	
$\pi\pi$	1346	-275	1405	-74	$-,-,-$	VI	$\leftarrow f_0(1370)?$
			1445	-116	$-,+,+$	II	
			1424	-94	$-,+,-$	VII	
			1456	-47	$-,-,+$	III	$\leftarrow f_0(1500)?$
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			159	0	$-,-,-$	VI	
			418	-10	$-,-,+$	III	
			1038	-204	$-,+,-$	VII	
			988	-31	$-,+,+$	II	$\leftarrow f_0(980)$
$\sigma\sigma$	118	-2227	4741	-4688	$-,-,-$	VI	
			3687	-2875	$-,+,-$	VII	
			3626	-3456	$+,-,-$	V	
			3533	-579	$+,+,-$	VIII	

$\rho(770)$

$I^G(J^{PC}) = 1^+(1^{--})$

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ρ(770) MASS

We no longer list *S*-wave Breit-Wigner fits, or data with high comb background.

NEUTRAL ONLY, e^+e^-

VALUE (MeV)	*EVTS*	*DOCUMENT ID*		*TECN*	*COMM*
775.26±0.25 OUR AVERAGE					
775.02±0.35		[1] LEES	12G	BABR	e^+e^-
775.97±0.46±0.70	900k	[2] AKHMETSHIN	07		e^+e^-
774.6 ±0.4 ±0.5	800k	[3,4] ACHASOV	06	SND	e^+e^-
775.65±0.64±0.50	114k	[5,6] AKHMETSHIN	04	CMD2	e^+e^-
775.9 ±0.5 ±0.5	1.98M	[7] ALOISIO	03	KLOE	1.02 e π^+
775.8 ±0.9 ±2.0	500k	[7] ACHASOV	02	SND	1.02 e π^+
775.9 ±1.1		[8] BARKOV	85	OLYA	e^+e^-
• • • We do not use the following data for averages, fits, limits, etc. • •					
775.8 ±0.5 ±0.3	1.98M	[9] ALOISIO	03	KLOE	1.02 e π^+
775.9 ±0.6 ±0.5	1.98M	[10] ALOISIO	03	KLOE	1.02 e π^+
775.0 ±0.6 ±1.1	500k	[11] ACHASOV	02	SND	1.02 e π^+
775.1 ±0.7 ±5.3		[12] BENAYOUN	98	RVUE	e^+e^- μ^+
770.5 ±1.9 ±5.1		[13] GARDNER	98	RVUE	0.28–(π^+
764.1 ±0.7		[14] O'CONNELL	97	RVUE	e^+e^-
757.5 ±1.5		[15] BERNICHA	94	RVUE	e^+e^-
768 ±1		[16] GESHKEN...	89	RVUE	e^+e^-

CHARGED ONLY, τ DECAYS and e^+e^-

VALUE (MeV)	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*	*COM*
775.11±0.34 OUR AVERAGE						
774.6 ±0.2 ±0.5	5.4M	[17,18] FUJIKAWA	08	BELL	±	τ^-
775.5 ±0.7		[18,19] SCHAEL	05C	ALEP		τ^-
775.5 ±0.5 ±0.4	1.98M	[7] ALOISIO	03	KLOE		1.02 π
775.1 ±1.1 ±0.5	87k	[20,21] ANDERSON	00A	CLE2		τ^-
• • • We do not use the following data for averages, fits, limits, etc. • •						
774.8 ±0.6 ±0.4	1.98M	[10] ALOISIO	03	KLOE	−	1.02 π
776.3 ±0.6 ±0.7	1.98M	[10] ALOISIO	03	KLOE	+	1.02 π
773.9 ±2.0 $^{+0.3}_{-1.0}$		[22] SANZ-CILLERO	03	RVUE		τ^-
774.5 ±0.7 ±1.5	500k	[7] ACHASOV	02	SND	±	1.02 π
775.1 ±0.5		[23] PICH	01	RVUE		τ^-

MIXED CHARGES, OTHER REACTIONS

VALUE (MeV)	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
763.0±0.3±1.2	600k	[24] ABELE	99E	CBAR	0±	0.0 $\bar{p}p \to \pi^+\pi^-\pi^0$

CHARGED ONLY, HADROPRODUCED

VALUE (MeV)	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
766.5±1.1 OUR AVERAGE						
763.7±3.2		ABELE	97	CBAR		$\bar{p}n \to \pi^-\pi^0\pi^0$
768 ±9		AGUILAR-...	91	EHS		400 pp
767 ±3	2935	[25] CAPRARO	87	SPEC	−	200 π^-Cu $\to \pi^-\pi^0$Cu
761 ±5	967	[25] CAPRARO	87	SPEC	−	200 π^-Pb $\to \pi^-\pi^0$Pb
771 ±4		HUSTON	86	SPEC	+	202 $\pi^+A \to \pi^+\pi^0A$
766 ±7	6500	[26] BYERLY	73	OSPK	−	5 π^-p
766.8±1.5	9650	[27] PISUT	68	RVUE	−	1.7–3.2 π^-p, $t<10$
767 ±6	900	[25] EISNER	67	HBC	−	4.2 π^-p, $t<10$

NEUTRAL ONLY, PHOTOPRODUCED

VALUE (MeV)	*EVTS*	*DOCUMENT ID*		*TECN*	*COMMENT*
769.0± 1.0 OUR AVERAGE					
771 ± 2 $^{+2}_{-1}$	63.5k	[28] ABRAMOWICZ	12	ZEUS	$ep \to e\pi^+\pi^-p$
770 ± 2 ±1	79k	[29] BREITWEG	98B	ZEUS	50–100 γp
767.6± 2.7		BARTALUCCI	78	CNTR	$\gamma p \to e^+e^-p$
775 ± 5		GLADDING	73	CNTR	2.9–4.7 γp
767 ± 4	1930	BALLAM	72	HBC	2.8 γp
770 ± 4	2430	BALLAM	72	HBC	4.7 γp
765 ±10		ALVENSLEB...	70	CNTR	γA, $t<0.01$
767.7± 1.9	140k	BIGGS	70	CNTR	<4.1 γC $\to \pi^+\pi^-$C
765 ± 5	4000	ASBURY	67B	CNTR	γ + Pb
• • • We do not use the following data for averages, fits, limits, etc. • • •					
771 ± 2	79k	[30] BREITWEG	98B	ZEUS	50–100 γp

NEUTRAL ONLY, OTHER REACTIONS

VALUE (MeV)	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
769.0±0.9 OUR AVERAGE	Error includes scale factor of 1.4. See the ideogram below.					
765 ±6		BERTIN	97C	OBLX		0.0 $\bar{p}p \to \pi^+\pi^-\pi^0$
773 ±1.6		WEIDENAUER	93	ASTE		$\bar{p}p \to \pi^+\pi^-\omega$
762.6±2.6		AGUILAR-...	91	EHS		400 pp
770 ±2		[31] HEYN	81	RVUE		Pion form factor
768 ±4		[32,33] BOHACIK	80	RVUE	0	

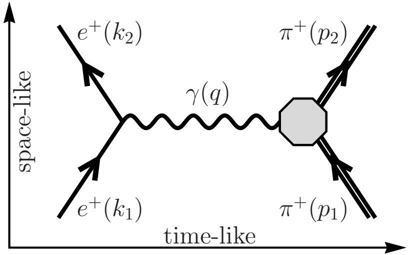
Pion electromagnetic form factor in the P wave

What are the correct $\rho(770)$ meson mass and width values?

PRD 96, 113004 (2017)

Erik Bartoŝ, Stanislav Dubniĉka, Andrej Liptaj Anna Zuzana Dubniĉkova and RK

$$\langle \pi^+(p_2)|J^\mu_\pi(0)|\pi^-(p_1)\rangle = e\,(p_1+p_2)^\mu\, F_\pi(q^2)$$



Gounaris-Sakurai pion EM FF and $\rho(770)$

Gounaris-Sakurai pion electromagnetic form factor at the elastic region.

What are the correct $\rho(770)$ *meson mass and width values?*
Erik Bartoŝ, Stanislav Dubniĉka, Andrej Liptaj Anna Zuzana Dubniĉkova and RK
PRD 96, 113004 (2017)

$$\sigma_{tot}(e^+e^- \to \pi^+\pi^-) = \frac{\pi\alpha^2(0)}{3s}\beta_\pi^3(s)\left|F_\pi^{EM,I=1}(s) + Re^{i\phi}\frac{m_\omega^2}{m_\omega^2 - s - im_\omega\Gamma_\omega}\right|^2$$

where pion "velocity" $\beta_\pi(s) = \sqrt{\frac{s-4m_\pi^2}{s}}$, R - amplitude for $\rho - \omega$ interference (free parameter), phase $\phi = ArcTan\frac{m_\rho\Gamma_\rho}{m_\rho^2 - m_\omega^2}$ is the $\delta_1^1 = \delta_\rho$ fixed at $s = m_\omega^2$

Fit to data for $\sigma_{tot}(e^+e^- \to \pi^+\pi^-)$
- M. Ablikin et al. (BESIII Collaboration), Phys. Lett. B 753, 629 (2016).
- J. P. Lees et al. (BABAR Collaboration), Phys. Rev. D 86, 032013 (2012).

Gounaris-Sakurai pion electromagnetic form factor

G. J. Gounaris and J. J. Sakurai, Phys. Rev. Lett. 21, 244 (1968)

Assumption: $\frac{q^3}{\sqrt{s}} Cotg\delta_1^1(s) = a + bq^2 + h(s)q^2$ where $h(s) = \frac{2q}{\pi\sqrt{s}} Log\left(\frac{\sqrt{s}+2q}{2m_\pi}\right)$
Then $F_\pi^{GS}(s) = \frac{\sqrt{s}}{q^3} \frac{1}{Cotg\delta_1^1(s)-i}$ - no dependence on $\rho(770)$, however taking into account two conditions:

- $Cotg\delta_1^1(s)\Big|_{s=m_\rho^2} = 0$ and
- $F_\pi^{BW}(s) = \frac{m_\rho^2}{m_\rho^2-s-im_\rho\Gamma_\rho} \to \delta_1^1(s) = ArcTan\frac{m_\rho\Gamma_\rho}{m_\rho^2-s} \to \frac{d\delta_1^1(s)}{ds}\Big|_{s=m_\rho^2} = \frac{1}{m_\rho\Gamma_\rho}$

$$a = \frac{4q_\rho^5}{m_\rho^2\Gamma_\rho} + 4q_\rho^4 h'(m_\rho^2)$$

$$b = -\frac{4q_\rho^3}{m_\rho^2\Gamma_\rho} - 4q_\rho^4 h'(m_\rho^2) - h(m_\rho^2)$$

$$F_\pi^{GS}(s) = \frac{m_\rho^2+m_\rho\Gamma_\rho\left[\frac{3m_\pi^2}{\pi q_\rho^2} log\left(\frac{m_\rho+2mq_\rho}{2m_\pi}\right)+\frac{m_\rho}{2\pi q_\rho}-\frac{m_\pi^2 m_\rho}{\pi q_\rho^3}\right]}{(m_\rho^2-s)+\Gamma_\rho\frac{m_\rho^2}{q_\rho^3}\left[q^2\left(h(s)-h(m_\rho^2)\right)+q_\rho^2 h'(m_\rho^2)(m_\rho^2-s)\right]-im_\rho\Gamma_\rho\left(\frac{q}{q_\rho}\right)^3\frac{m_\rho}{\sqrt{s}}}$$

Fit to unified BESIII-BABAR data at the elastic region using G-S model

- $\chi^2 = 40.6$ pdf
- $m_\rho = (775.73 \pm 0.10)$ MeV
- $\Gamma_\rho = (126.51 \pm 0.13)$ MeV
- PDG'2017:

  $m_\rho = 775.26 \pm 0.25$ MeV (from $e^+e^- \rightarrow \pi^+\pi^-$)
  $m_\rho = 769.0 \pm 0.9$ MeV (other)

  $\Gamma_\rho = 147.8 \pm 0.9$ MeV (from $e^+e^- \rightarrow \pi^+\pi^-$)
  $\Gamma_\rho = 150.9 \pm 1.7$ MeV (other)

in Ref. [6].

Now, the $\rho^0(770)$ meson parameters will be determined by an application of the U&A model of the pion EM FF to an optimal description of the same data on the total cross section of the $e^+e^- \rightarrow \pi^+\pi^-$ process.

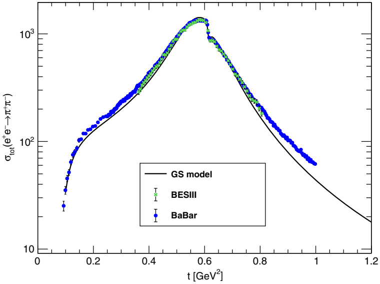


FIG. 2. Optimal description of the unified BESIII-*BABAR* data on $\sigma_{\text{tot}}(e^+e^- \rightarrow \pi^+\pi^-)$ at the elastic region by the pion EM FF G.-S. model.

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Fit to unified BESIII-BABAR data at the elastic region using U&A model

- $F_\pi^{VDM}(s) = \frac{m_\rho^2 f_{\rho\pi\pi}/f_\rho}{m_\rho^2 - s}$
- $F_\pi^{VDM}(q) = \frac{(i-q_\rho)(i+q_\rho)}{(q-q_\rho)(q+q_\rho)} f_{\rho\pi\pi}/f_\rho$
- $F_\pi^{VDM}(q) = \frac{(q-q_Z)(i-q_P)(i-q_\rho)(i+q_\rho)}{(q-q_P)(i-q_Z)(q-q_\rho)(q+q_\rho)} f_{\rho\pi\pi}/f_\rho$
- $\chi^2 = 1.54$ pdf
- $m_\rho = (763.03 \pm 0.14)$ MeV
- $\Gamma_\rho = (144.8 \pm 0.23)$ MeV

The optimal description of the recent data [1,2] on the total cross section of the $e^+e^- \to \pi^+\pi^-$ process at the region of the ρ meson resonance by (21) (see Fig. 3), achieved with $\chi^2/ndf = 1.5443$, gives the ρ meson mass and width

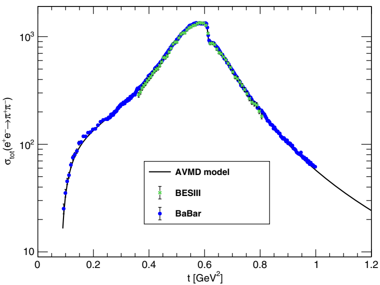
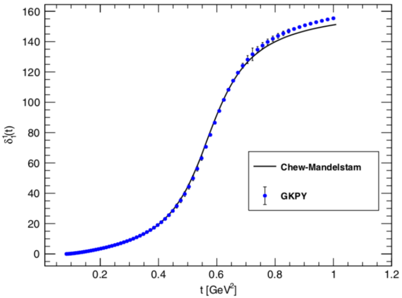


FIG. 3. Optimal description of the unified BESIII-*BABAR* data on $\sigma_{tot}(e^+e^- \to \pi^+\pi^-)$ at the elastic region by the pion EM FF U&A model.

Fit to δ_1^1 data using Chew-Mandelstam type effective-range formula

- $\frac{q^3}{\sqrt{s}} Cotg\delta_1^1(s) = a + bq^2 + h(s)q^2$
  where $h(s) = \frac{2q}{\pi\sqrt{s}} Log\left(\frac{\sqrt{s}+2q}{2m_\pi}\right)$
- $a = 0.2860 \pm 0.0011$ MeV^2,
  $b = -2.7025 \pm 0.0089$
- $\chi^2 = 2.45$ pdf
- $m_\rho = (772.42 \pm 0.03)$ MeV
- $\Gamma_\rho = (153.85 \pm 0.11)$ MeV



Fit to δ_1^1 data

- $F_\pi^{EM,I=1}(s) =$

$$P_n(s)exp\left[\frac{s}{\pi}\int_4^\infty \frac{\delta_1^1(s')}{s'(s'-s)}ds'\right]$$

- $\delta_1^1(q) = ArcTan\frac{A_3q^3+A_5q^5+\dots}{1+A_2q^2+A_4q^4+\dots}$
- $\equiv Log\frac{(q-q_2)(q-q_3)(q-q_4)(q-q_5)}{(q-q_2^*)(q-q_3^*)(q-q_4^*)(q-q_5^*)}$
- $\chi^2 = 0.024$ pdf
- $m_\rho = (763.56 \pm 0.51)$ MeV
- $\Gamma_\rho = (143.09 \pm 0.82)$ MeV

$$\delta_1^1(q) = arctg\frac{A_3q^3+A_5q^5+\cdots}{1+A_2q^2+A_4q^4+\cdots}, \quad (26)$$

where $A_1 \equiv 0$ in order to secure the threshold behavior of $\delta_1^1(q)$. An optimal description of the GKPY phase shift $\delta_1^1(q)$ data is achieved (see Fig. 5) with $\chi^2/ndf = 0.0244$ and four nonzero coefficients A_2, A_3, A_4, and A_5.

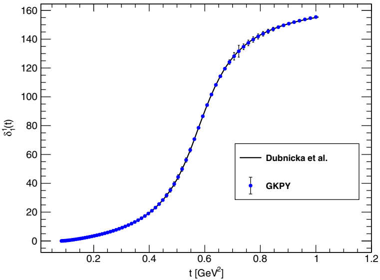


FIG. 5. Optimal description of the most accurate up-to-now P-wave isovector $\pi\pi$ scattering phase shift $\delta_1^1(t)$ data with model-independent parametrization (26).

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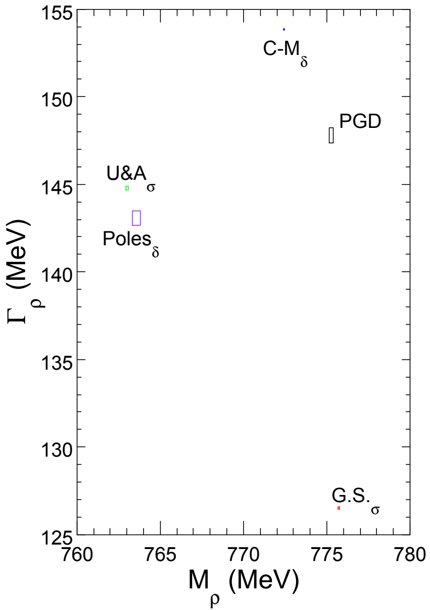
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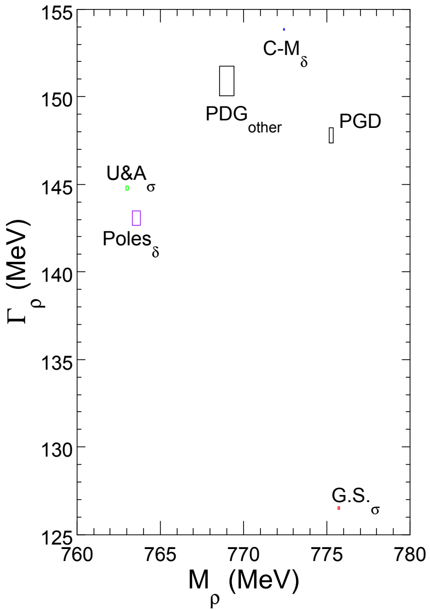
Γ_ρ (MeV)
M_ρ (MeV)
C-M$_\delta$
PGD
U&A$_\sigma$
Poles$_\delta$
G.S.$_\sigma$



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Polesδ
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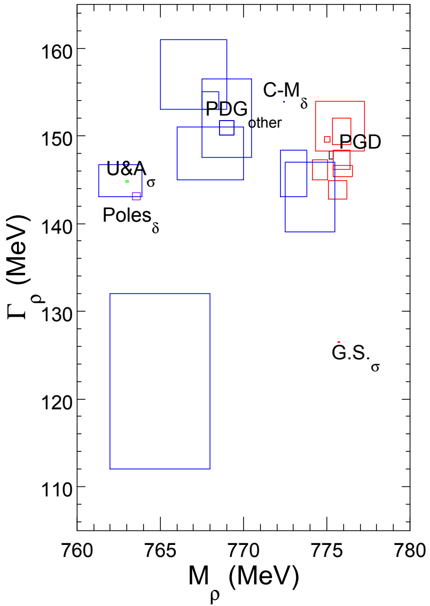
Mρ (MeV)



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M_ρ (MeV)
Γ_ρ (MeV)
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other
C-M
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PGD
U&A
σ
Poles
δ
G.S.
σ

Generalisation of Gounaris-Sakurai model to excited ρ mesons $\rho(1450)$ and $\rho(1700)$

- $F_\pi = \frac{1}{1+\beta+\gamma}\left[F^{GS}_{\rho(770)}(s)\left(1 + \delta \frac{s}{m^2_\omega} BW_\omega(s)\right) + \beta F^{GS}_{\rho(1450)}(s) + \gamma F^{GS}_{\rho(1700)}(s)\right]$
- $\chi^2 = 0.98$ pdf

Generalisation of U&A model to excited ρ mesons

- $F_\pi = \frac{\Pi(q-q_i)}{\Pi(q+q^*_i)}$
- $\chi^2 = 1.84$ pdf

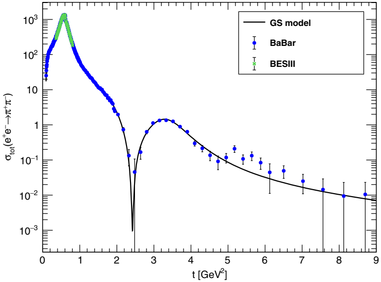

FIG. 6. Optimal description of the unified BESIII-*BABAR* complete data on $\sigma_{\rm tot}(e^+e^- \to \pi^+\pi^-)$ by the generalized pion EM FF G.-S. model.

determined by the original pion EM FF G.-S. model (13) to be valid only at the elastic region.

A totally different situation is in a generalization of the U&A pion EM FF model. Here, the contribution of all three vector mesons is at an equal level. Only now, the effective inelastic threshold, which is left as a free parameter of the model, has to be taken into account explicitly. Therefore, instead of the q variable, the W variable

Parameter	PDG MeV	G.S. MeV	U&A MeV
m_ρ	775.26 ± 0.25	774.81 ± 0.01	763.88 ± 0.04
$m_{\rho'}$	1465.00 ± 25.00	1497.70 ± 1.07	1326.35 ± 3.46
$m_{\rho''}$	1720.00 ± 20.00	1848.40 ± 0.09	1770.54 ± 5.49
Γ_ρ	149.10 ± 0.80	149.22 ± 0.01	144.28 ± 0.01
$\Gamma_{\rho'}$	400.00 ± 60.00	442.15 ± 0.54	324.13 ± 12.01
$\Gamma_{\rho''}$	250.00 ± 100.00	322.48 ± 0.69	268.98 ± 11.40
χ^2 pdf		0.98	1.84
		14 param.	11 param.

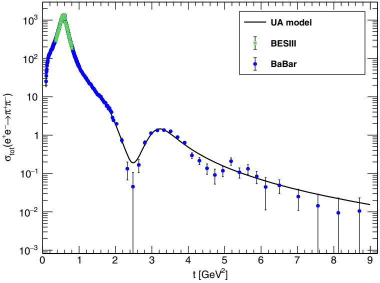

FIG. 7. Optimal description of the unified BESIII-*BABAR* complete data on $\sigma_{tot}(e^+e^- \to \pi^+\pi^-)$ by the generalized pion

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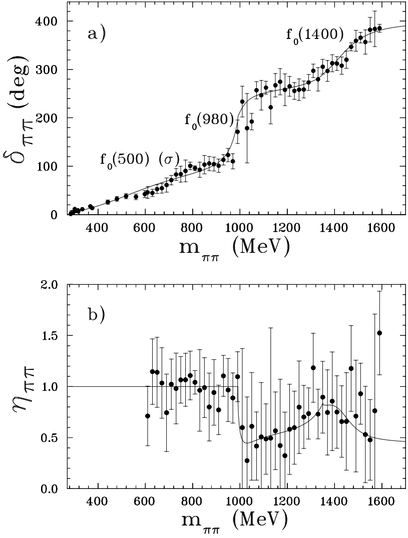
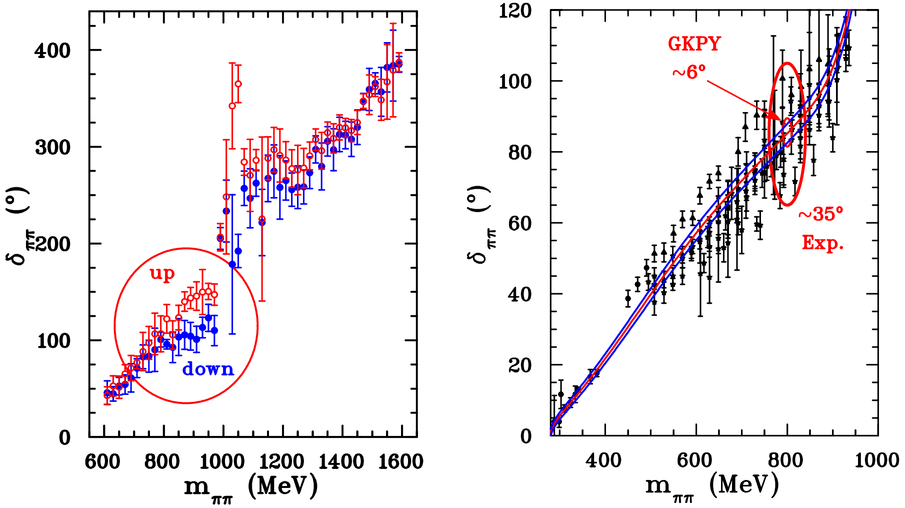
what is the meson $f_0(500)$?

- formally $f_0(500)$ (informally σ) with mass (before 2012): M : $400 - 1200$ MeV and width Γ : $500 - 1000$ MeV,
- the lightest scalar-isoscalar meson with $I^G J^{PC} = 0^+0^{++}$, decays into $\pi\pi$,
- had a rich but difficult life,
- very important for e.g.
    - calculation of quark condensate mass,
    - determination of $q\bar{q} - gg$ couplings,
    - parameterization of $\pi\pi$ S wave amplitudes in e.g. many heavy meson decays (FSI)
- difficult to study

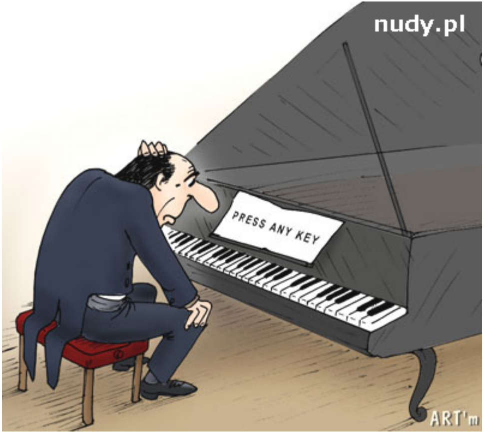


Experimental data for the $\pi\pi$ in the S0 wave (JI)

In PWA (CERN-Munich group'74) $A(s,t) \sim Cos(\theta_S - \theta_P)$

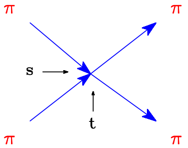


The pion-pion scattering amplitude'2008 (Roy eqs) ...



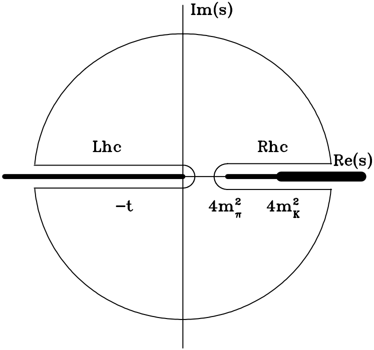
Dispersion relations with imposed crossing symmetry condition for $\pi\pi$ interactions theory $\longleftrightarrow$ experiment

crossing symmetry:



$\rightarrow \vec{T}_s(s,t) = \hat{C}_{st}\vec{T}_t(t,s)$

$\vec{T}(s,t)$ + crossing symmetry $\rightarrow$ dispersion relations for $\underline{4m_\pi^2 < s < \sim (1150\ \text{MeV})^2}$

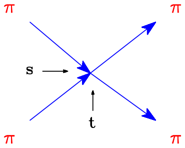


Once subtracted DR:

$$\text{Re}\,\vec{F}(s,t) = \text{Re}\,\vec{F}(s_0,t) + \frac{s-s_0}{\pi} \times \left[\int_{4m_\pi^2}^{\infty} ds' \frac{\text{Im}\,\vec{F}(s',t)}{(s'-s_0)(s'-s)} + \int_{-t}^{-\infty} ds' \frac{\text{Im}\,\vec{F}(s',t)}{(s'-s_0)(s'-s)} \right]$$

Dispersion relations with imposed crossing symmetry condition for $\pi\pi$ interactions theory $\longleftrightarrow$ experiment

crossing symmetry:



$\rightarrow \vec{T}_s(s,t) = \hat{C}_{st}\vec{T}_t(t,s)$

$\vec{T}(s,t)$ + crossing symmetry $\rightarrow$ dispersion relations for $4m_\pi^2 < s < \sim (1150\text{ MeV})^2$

Once subtracted dispersion relations ("GKPY" for the S and P waves):

$$\text{Re } t_\ell^{I(OUT)}(s) = \sum_{I'=0}^{2} C_{st}^{II'} a_0^{I'} + \sum_{I'=0}^{2}\sum_{\ell'=0}^{4} \int_{4m_\pi^2}^{\infty} ds' K_{\ell\ell'}^{II'}(s,s') \,\text{Im } t_{\ell'}^{I'(IN)}(s')$$

$a_0^{I'}$ - subtraction constant $= \vec{T}_s(s=4m_\pi^2, t=0)$ - scattering lengths from only S wave

due to $\text{Re } t_\ell^I(k) = k^{2\ell}(a_\ell^I + b_\ell^I k^2 + O(k^4))$ $\quad\quad$ $\text{Re } t_\ell^{I(OUT)}(s) - \text{Re } t_\ell^{I(IN)}(s) \rightarrow 0$

GKPY equations and $\pi\pi$ amplitudes

partial waves: *Jl*

experiment

F1 **D2**

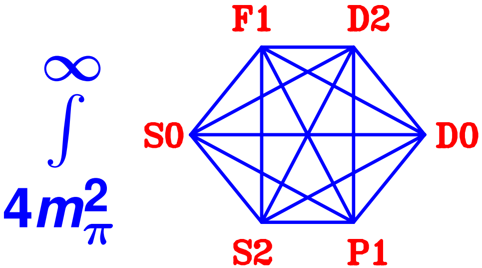
S0 **D0**

S2 **P1**

GKPY equations and $\pi\pi$ amplitudes

partial waves: *Jl*

experiment + theory (GKPY)



Experimental data for the $\pi\pi$ in the S0 wave (JI)

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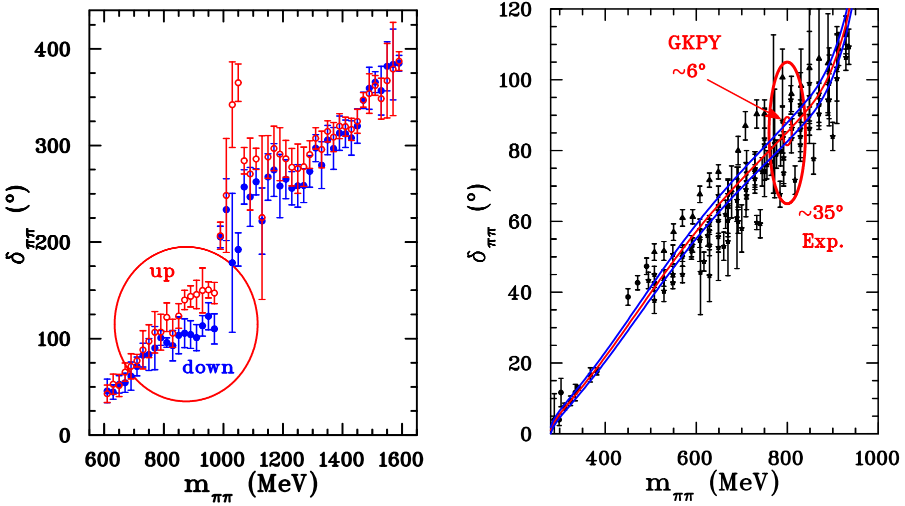
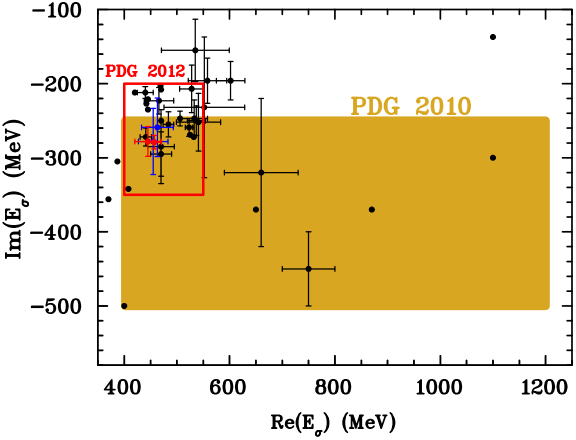


precise determination of $f_0(500)$ (σ) meson and threshold parameters

$f_0(500)$ (σ)

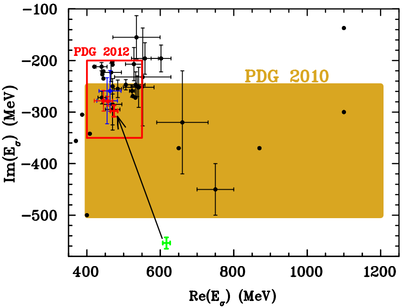
- PDG 2010:
  M = 400 – 1200 MeV
  $\Gamma = 2 \times (250 - 500)$ MeV
- PDG 2012:
  M = 400 – 550 MeV
  $\Gamma = 2 \times (200 - 350)$ MeV
- GKPY:
  $E_\sigma = 457 \pm 14 - i279^{+11}_{-7}$ MeV



threshold parameters, e.g. a_0^0:

- ChPT + Roy eqs (Bern group):
  $0.220 \pm 0.005\ m_\pi^{-1}$
- GKPY:
  $0.220 \pm 0.008\ m_\pi^{-1}$

what forces GKPY eqs to pull up-left the sigma pole?



Two things: trigonometry and crossing symmetry algebra lead to narrower and lighter σ.

Modified $\pi\pi$ amplitude with σ pole PRD 90, 116005 (2014) P. Bydzovský, 1, R. Kamiński, V. Nazari

Nothing more and nothing instead of it is needed.

Resonance is near the threshold

1976 S. M. Flatté analyses the $\pi\eta$ and the $K\bar{K}$ coupled channel systems

$$A_i \sim \frac{M_R\sqrt{\Gamma_0\Gamma_i}}{M_R^2 - E^2 - iM_R(\Gamma_1+\Gamma_2)}, \quad i = 1, 2.$$

$\Gamma_i = g_i k_i$ and $\Gamma_0 = g_1 q$ with $q = k_1(2m_K)$. So THREE free parameters: M_R, g_1, g_2.

One channel case:

$$T_{22} = \frac{\sin\delta_2}{k_2} e^{i\delta_2} \equiv \frac{1}{k_2\cot\delta_2 - ik_2},$$

$$k_2\cot\delta_2 \approx \frac{1}{a} + \frac{1}{2} r\, k_2^2 \longrightarrow T_{22} = \frac{1}{\frac{1}{a} - i\,k_2 + \frac{1}{2}\, r\, k_2^2}$$

where a is the scattering length and r is the effective range (both real).
Two channel case: A and R are complex so FOUR free parameters

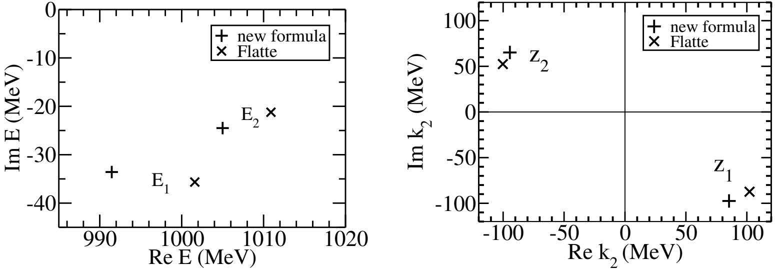
$$T_{22} = \frac{1}{2ik_2}(\eta e^{2i\delta_2} - 1) \longrightarrow T_{22} = \frac{1}{\frac{1}{A} - i\,k_2 + \frac{1}{2}\, R\, k_2^2}.$$

$$A = -i(\frac{1}{z_1} + \frac{1}{z_2}), \qquad R = \frac{2i}{z_1 + z_2}\,.$$

where z_1 and z_2 are zeroes of the S_{22} matrix element and are related with resonance.
Flatte approach: $ImR = 0$ so $Rez_1 = -Rez_2$

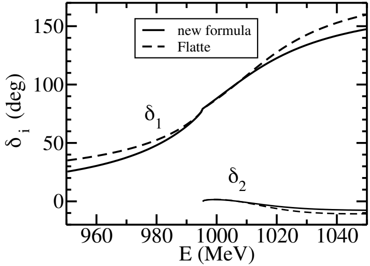
For $a_0(980)$

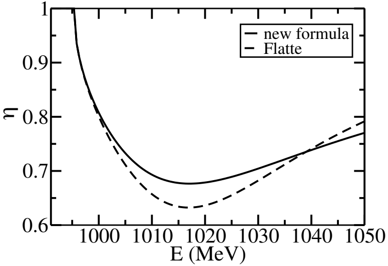
L. Leśniak, AIP Conf.Proc. 1030 (2008) 238



For $a_0(980)$

L. Leśniak, Int.J.Mod.Phys. A24 (2009) 549





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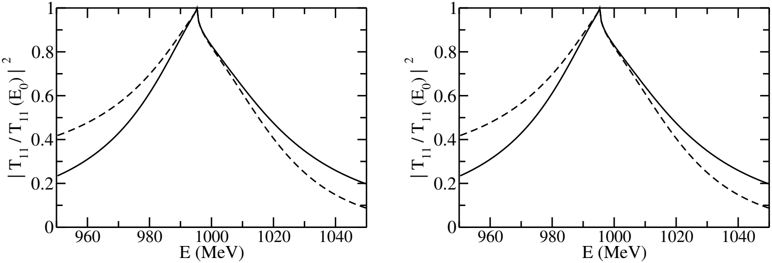

Conclusions and what to do?

Conclusions:

- there is significant difference between parameters (especially mass) of a resonance using different methods of parameterisation of amplitudes,
- amplitudes should be unitary and analytic
- we should always determine what definition we use,
- the determination of the most significant poles in amplitudes becomes difficult for $n > 2$,

What to do?

- never use isobar model!
- anyway be carefull! = check the 2nd point of conclusions!
- possible parameterizations: made for wide energy range including many resonances we are interested in,
- please use only parameterization based on S- or K-matrix approach!
- possible degrees of freedom: poles themself! mesons
- on $\pi\pi$, $K\bar{K}$ ... amplitudes in all partial waves up to 1.8 GeV ask RK - they are done
- on other channels ask RK - they may be done

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- on $\pi\pi$, $K\bar{K}$ ... amplitudes in all partial waves up to 1.8 GeV ask RK - they are done
- on other channels ask RK - they may be done

Conclusions on scalar resonances

Experimental situation in the 0^+0^{++} sector is finally clear (although without new experiments!),

- $f_0(500)$ (σ) - loosely bound $\pi\pi$ system or/and $2q2\bar{q}$ state,
- $f_0(980)$ - dynamically generated $2q2\bar{q}$ state or/and $K\bar{K}$ quasi bound state,
- $f_0(1370)$ - ordinary $q\bar{q}$ state,
- $f_0(1500)$ - ordinary $q\bar{q}$ state + glueball,
- $f_0(1720)$ - glueball

Probably $K_0^*(800)$ (κ) is also dynamical effect of coupling of $K_0^*(1430)$