# A covariant approach to the partial wave analysis of the hadron reactions

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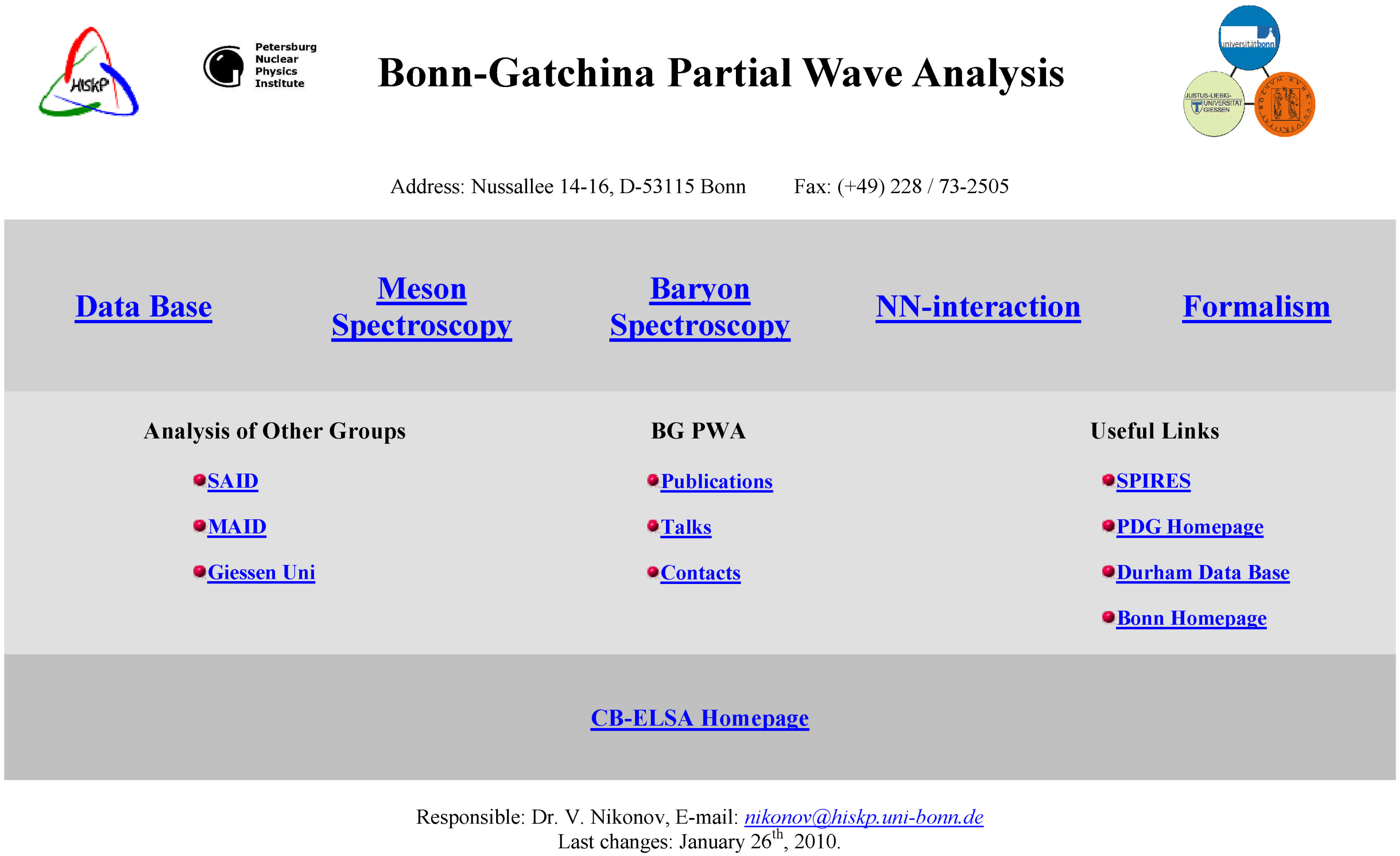
**http://pwa.hiskp.uni-bonn.de/**



# Search for baryon states

1. **Analysis of single and double meson photoproduction reactions.**
   $\gamma p \to \pi N, \eta N, K\Lambda, K\Sigma, \pi\pi N, \pi\eta N$**,** $\omega p$**,** $K^*\Lambda$**,**
   **CB-ELSA, CLAS, MAMI, GRAAL, LEPS.**

2. **Analysis of single and double meson production in pion-induced reactions.**
   $\pi N \to \pi N, \eta N, K\Lambda, K\Sigma, \pi\pi N$**.**

# Search for meson states

1. **Analysis of the** $p\bar{p}$ **annihilation at rest and** $\pi\pi$ **interaction data.**

2. **Analysis of the** $p\bar{p}$ **annihilation in flight into two and tree meson final state.**

3. **Analysis of the BES III data on** $J/\Psi$ **decays (in collaboration with JINR Dubna).**

# Analysis of $NN$ interaction

1. **Analysis of single and double meson production** $NN \to \pi NN$ **and (Wasa, PNPI, HADES)**

2. **Analysis of hyperon production** $NN \to K\Lambda p$ **(WASA, HADES)**

# Energy dependent approach

**In many cases an unambiguous partial wave decomposition at fixed energies is impossible. Then the energy and angular parts should be analyzed together:**

$$A(s,t) \ = \ \sum_{\beta\beta' n} A_n^{\beta\beta'}(s) Q^{(\beta)+}_{\mu_1\ldots\mu_n} F^{\mu_1\ldots\mu_n}_{\nu_1\ldots\nu_n} Q^{(\beta')}_{\nu_1\ldots\nu_n}$$

$A_n^{\beta\beta'}(s)$ **- the partial wave amplitude with total spin** $J=n$ **for bosons and** $J=n+1/2$ **for fermions.**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1. C. Zemach, Phys. Rev. 140, B97 (1965); 140, B109 (1965).
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# Partial wave amplitude:

## transition amplitude with fixed initial and final states

**Quantum numbers: mesons $I^G\,J^{PC}$, baryons: $I\,J^P$, decay LS basis: $^{2S+1}L_J$**

$$I_1^{G_1}J_1^{P_1C_1}+I_2^{G_2}J_2^{P_2C_2}\left(^{2S+1}L_J\right)\to I^G J^{PC}\to I_1'^{G_1'}J_1'^{P_1'C_1'}+I_2'^{G_2'}J_2'^{P_2'C_2'}\left(^{2S'+1}L_J'\right)$$

$$G=G_1G_2 \qquad G=G_1'G_2'$$

$$P=P_1P_2(-1)^L \qquad P=P_1'P_2'(-1)^{L'}$$

$$|I_1-I_2|<I<I_1+I_2 \qquad |I_1'-I_2'|<I<I_1'+I_2'$$

$$|J_1-J_2|<S<J_1+J_2 \qquad |J_1'-J_2'|<S'<J_1'+J_2'$$

$$|S-L|<J<S+L \qquad |S'-L'|<J<S'+L'$$

$$A(s,t)=V_{\mu_1\ldots\mu_n}(S,L)\,P^{\mu_1\ldots\mu_n}_{\nu_1\ldots\nu_n}\,V'_{\nu_1\ldots\nu_n}(S',L')A(s)$$

$n=J$ **mesons** $\qquad n=J-1/2$ **baryons**

# Boson projection operators

**The wave function of boson with $J=n$:**

$$\Psi_{\mu_1\cdots\mu_n} = \frac{1}{\sqrt{2\varepsilon}} u_{\mu_1\cdots\mu_n} e^{ipx}$$

$$\int \Psi_\mu(x)\Psi^*(x)d^4x = \alpha p_\mu = 0 \qquad \Longrightarrow \qquad p_\mu \Psi_\mu = 0$$

$$\int \Psi_{\mu\nu}(x)\Psi^*(x)d^4x = \beta\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) = 0 \qquad \Longrightarrow \qquad g_{\mu\nu}\Psi_{\mu\nu} = 0$$

**Properties of $u_{\mu_1\cdots\mu_n}$:**

$$p^2 u_{\mu_1\mu_2\ldots\mu_n} = m^2 u_{\mu_1\mu_2\ldots\mu_n}$$

$$p_{\mu_i} u_{\mu_1\mu_2\ldots\mu_n} = 0$$

$$g_{\mu_i\mu_j} u_{\mu_1\mu_2\ldots\mu_n} = 0$$

$$u_{\mu_1\ldots\mu_i\ldots\mu_j\ldots\mu_n} = u_{\mu_1\ldots\mu_j\ldots\mu_i\ldots\mu_n}$$

## In momentum representation:

$$P^{\mu_1\mu_2\ldots\mu_n}_{\nu_1\nu_2\ldots\nu_n} = (-1)^n O^{\mu_1\mu_2\ldots\mu_n}_{\nu_1\nu_2\ldots\nu_n} = \sum_{i=1}^{2n+1} u^{(i)}_{\mu_1\mu_2\ldots\mu_n} u^{(i)*}_{\nu_1\nu_2\ldots\nu_n}$$

$$O = 1$$

$$O^{\mu}_{\nu} = g^{\perp}_{\mu\nu} = g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}$$

$$O^{\mu_1\mu_2}_{\nu_1\nu_2} = \frac{1}{2}\left(g^{\perp}_{\mu_1\nu_1} g^{\perp}_{\mu_2\nu_2} + g^{\perp}_{\mu_1\nu_2} g^{\perp}_{\mu_2\nu_1}\right) - \frac{1}{3} g^{\perp}_{\mu_1\mu_2} g^{\perp}_{\nu_1\nu_2}$$

## Recurrent expression for the boson projector operator

$$O^{\mu_1\ldots\mu_L}_{\nu_1\ldots\nu_L} = \frac{1}{L^2}\left(\sum_{i,j=1}^{L} g^{\perp}_{\mu_i\nu_j} O^{\mu_1\ldots\mu_{i-1}\mu_{i+1}\ldots\mu_L}_{\nu_1\ldots\nu_{j-1}\nu j+1\ldots\nu_L} - \right.$$

$$\left. \frac{4}{(2L-1)(2L-3)} \sum_{i<j,k<m}^{L} g^{\perp}_{\mu_i\mu_j} g^{\perp}_{\nu_k\nu_m} O^{\mu_1\ldots\mu_{i-1}\mu_{i+1}\ldots\mu_{j-1}\mu_{j+1}\ldots\mu_L}_{\nu_1\ldots\nu_{k-1}\nu_{k+1}\ldots\nu_{m-1}\nu_{m+1}\ldots\nu_L}\right)$$

## Normalization condition:

$$O^{\mu_1\ldots\mu_L}_{\nu_1\ldots\nu_L} O^{\nu_1\ldots\nu_L}_{\alpha_1\ldots\alpha_L} = O^{\mu_1\ldots\mu_L}_{\alpha_1\ldots\alpha_L}$$

# Orbital momentum operator

**The angular momentum operator is constructed from momenta of particles $k_1$, $k_2$ and metric tensor $g_{\mu\nu}$.**

**For $L=0$ this operator is a constant: $X^0 = 1$**

**The $L=1$ operator is a vector $X^{(1)}_\mu$, constructed from: $k_\mu = \frac{1}{2}(k_{1\mu} - k_{2\mu})$ and $P_\mu = (k_{1\mu} + k_{2\mu})$. Orthogonality:**

$$\int \frac{d^4k}{4\pi} X^{(1)}_{\mu_1} X^{(0)} = \int \frac{d^4k}{4\pi} X^{(n)}_{\mu_1\ldots\mu_n} X^{(n-1)}_{\mu_2\ldots\mu_n} = \xi P_{\mu_1} = 0$$

**Then:**

$$X^{(1)}_\mu P_\mu = 0 \qquad\qquad X^{(n)}_{\mu_1\ldots\mu_n} P_{\mu_j} = 0$$

**and:**

$$X^{(1)}_\mu = k^\perp_\mu = k_\nu g^\perp_{\nu\mu}; \qquad\qquad g^\perp_{\nu\mu} = \left( g_{\nu\mu} - \frac{P_\nu P_\nu}{p^2} \right) \; ;$$

**in c.m.s** $k^\perp = (0, \vec{k})$

## Recurrent expression for the orbital momentum operators $X^{(n)}_{\mu_1\ldots\mu_n}$

$$X^{(n)}_{\mu_1\ldots\mu_n}=\frac{2n-1}{n^2}\sum_{i=1}^{n}k^{\perp}_{\mu_i}X^{(n-1)}_{\mu_1\ldots\mu_{i-1}\mu_{i+1}\ldots\mu_n}-\frac{2k^2_\perp}{n^2}\sum_{\substack{i,j=1\\ i<j}}^{n}g_{\mu_i\mu_j}X^{(n-2)}_{\mu_1\ldots\mu_{i-1}\mu_{i+1}\ldots\mu_{j-1}\mu_{j+1}\ldots\mu_n}$$

## Taking into account the traceless property of $X^{(n)}$ we have:

$$X^{(n)}_{\mu_1\ldots\mu_n}X^{(n)}_{\mu_1\ldots\mu_n}=\alpha(n)(k^2_\perp)^n \qquad \alpha(n)=\prod_{i=1}^{n}\frac{2i-1}{i}=\frac{(2n-1)!!}{n!}.$$

## From the recursive procedure one can get the following expression for the operator $X^{(n)}$:

$$X^{(n)}_{\mu_1\ldots\mu_n}=\alpha(n)\Big[k^\perp_{\mu_1}k^\perp_{\mu_2}\ldots k^\perp_{\mu_n}-\frac{k^2_\perp}{2n-1}\Big(g^\perp_{\mu_1\mu_2}k^\perp_{\mu_3}\ldots k^\perp_{\mu_n}+\cdots\Big)+$$
$$\frac{k^4_\perp}{(2n-1)(2n-3)}\Big(g^\perp_{\mu_1\mu_2}g^\perp_{\mu_3\mu_4}k^\perp_{\mu_5}\cdots k_{\mu_4}+\cdots\Big)+\cdots\Big].$$

## Scattering of two spinless particles

**Denote relative momenta of particles before and after interaction as $q$ and $k$, correspondingly. The structure of partial–wave amplitude with orbital momentum $L = J$ is determined by convolution of operators $X^{(L)}(k)$ and $X^{(L)}(q)$:**

$$A_L = BW_L(s) X^{(L)}_{\mu_1\ldots\mu_L}(k) O^{\mu_1\ldots\mu_L}_{\nu_1\ldots\nu_L} X^{(L)}_{\nu_1\ldots\nu_L}(q) = BW_L(s) X^{(L)}_{\mu_1\ldots\mu_L}(k) X^{(L)}_{\mu_1\ldots\mu_L}(q)$$

$BW_L(s)$ **depends on the total energy squared only.**

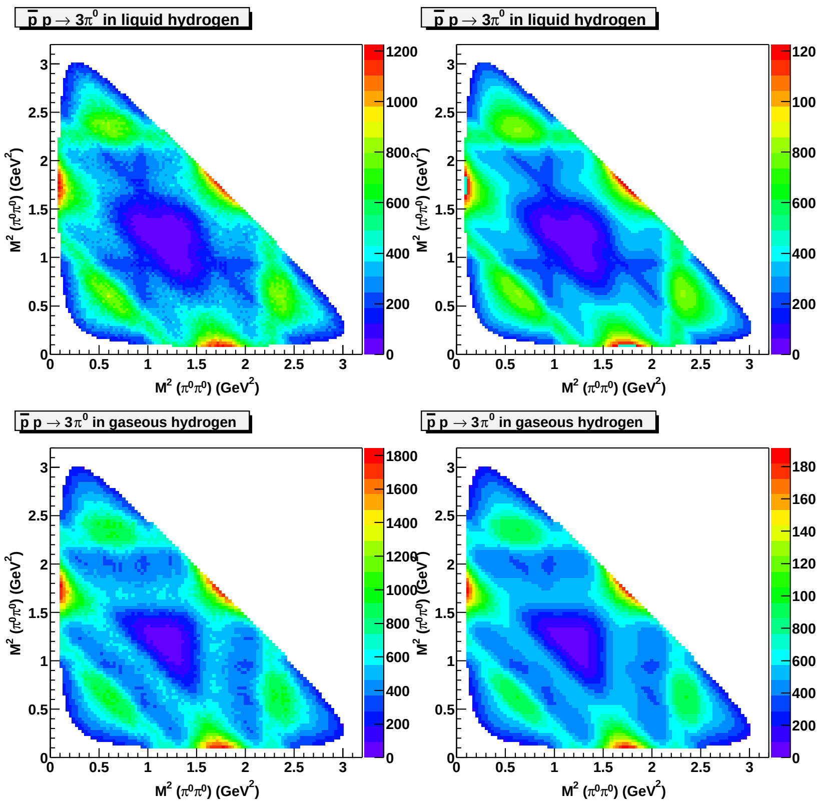
**The convolution $X^{(L)}_{\mu_1\ldots\mu_L}(k) X^{(L)}_{\mu_1\ldots\mu_L}(q)$ can be written in terms of Legendre polynomials $P_L(z)$:**

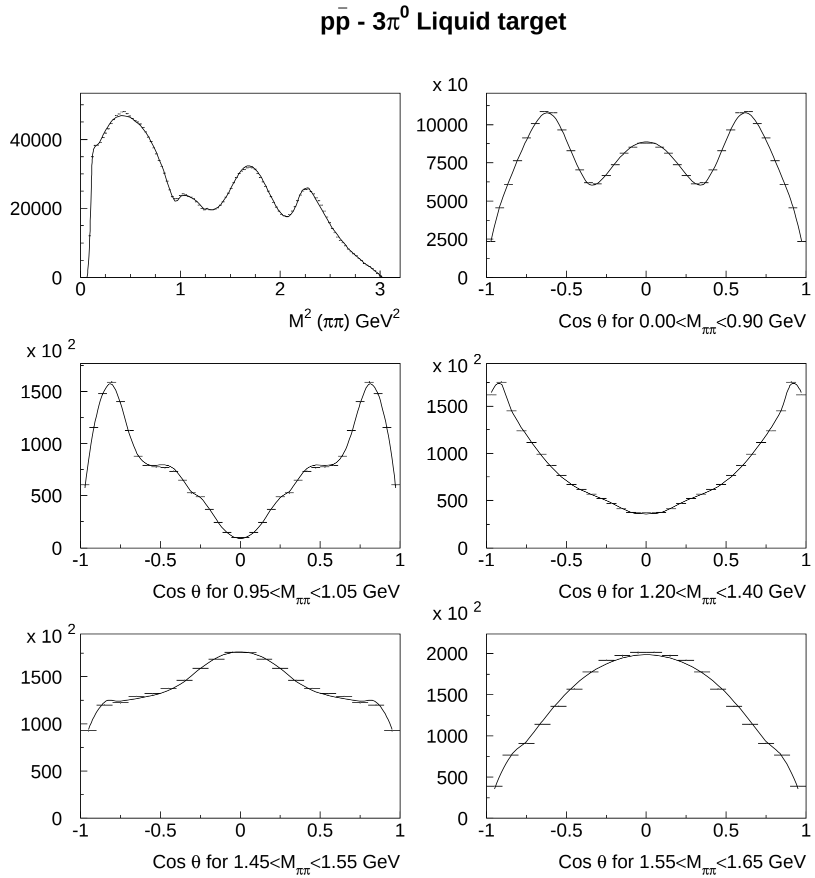
$$X^{(L)}_{\mu_1\ldots\mu_L}(k) X^{(L)}_{\mu_1\ldots\mu_L}(q) = \alpha(L) \left(\sqrt{k^2_\perp}\sqrt{q^2_\perp}\right)^L P_L(z) \,,$$

$$z = \frac{(k^\perp q^\perp)}{\sqrt{k^2_\perp}\sqrt{q^2_\perp}} \qquad\qquad \alpha(L) = \prod_{n=1}^{L} \frac{2n-1}{n}$$

# The $\bar{p}p \to 3\pi^0$ reaction

$$A(S-wave)=\sum_{L=0}^{N} X^{(L)}_{\mu_1\ldots\mu_L}(k^{\perp}_{23})X^{(L)}_{\mu_1\ldots\mu_L}(k^{\perp}_{1})A_L(s_{23})$$

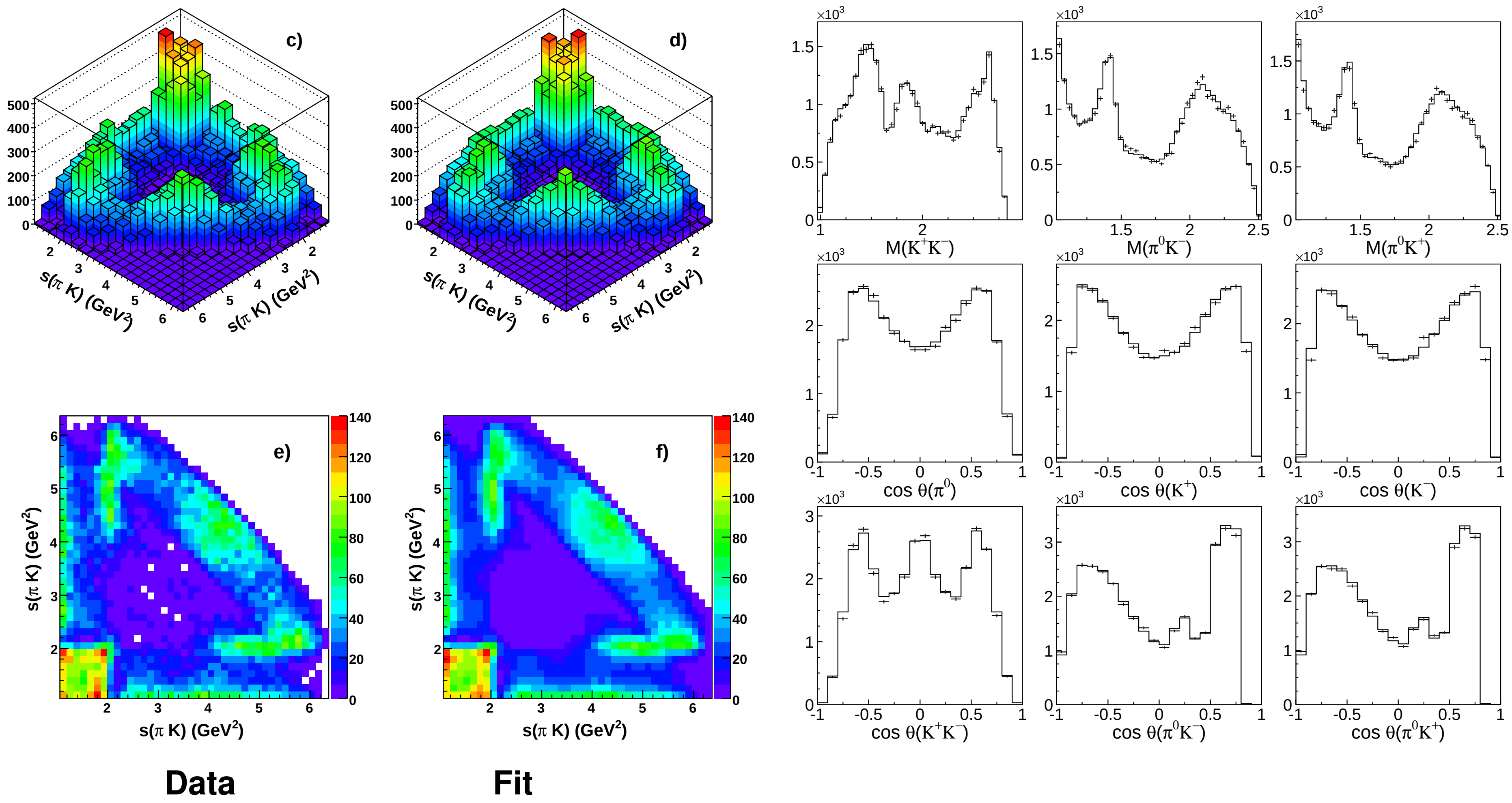


# $J/\Psi$ decay into three pseudoscalar mesons

$$A = V_\alpha A_\alpha \qquad \sum V^*_\mu V\nu = g^\perp_{\mu\nu} = g_{\mu\nu} - \frac{P_\mu P_\nu}{P^2}$$

$$A_\alpha = \varepsilon_{\alpha\eta\beta\mu} X_{\eta\nu_2\ldots\nu_J}(k_{23}) X_{\beta\nu_2\ldots\nu_J}(k_1^\perp) P_\mu A_J(s_{23}) \qquad k^\perp_{1\mu} = k_{1\nu} g^\perp_{\mu\nu}$$



# Structure of the fermion propagator

**The orthogonality condition has a different form in a fermion case:**

$$\int \Psi_\mu(x)\Psi^*(x)d^4x = A\, p_\mu + B\, \gamma_\mu$$

**where $A$ and $B$ are matrices in spinor space.**

**It means that we have an additional condition:**

$$\gamma_\mu \Psi_\mu = 0 \qquad\qquad \gamma_\mu u_\mu = 0$$

**Thus in momentum space we have:**

$$(\hat{p} - m)u_{\mu_1\ldots\mu_n} = 0$$

$$p_{\mu_i} u_{\mu_1\ldots\mu_n} = 0$$

$$u_{\mu_1\ldots\mu_i\ldots\mu_j\ldots\mu_n} = u_{\mu_1\ldots\mu_j\ldots\mu_i\ldots\mu_n}$$

$$g_{\mu_i\mu_j} u_{\mu_1\ldots\mu_n} = 0$$

$$\gamma_{\mu_i} \Psi_{\mu_1\ldots\mu_n} = 0$$

**These properties define the structure of the fermion projection operator $P^{\mu_1\ldots\mu_n}_{\nu_1\ldots\nu_n}$:**

$$G^{\mu_1\ldots\mu_n}_{\nu_1\ldots\nu_n} = \frac{(-1)^n}{2m}\frac{m+\hat{p}}{m^2-p^2}P^{\mu_1\ldots\mu_n}_{\nu_1\ldots\nu_n}$$

$$P^{\mu_1\ldots\mu_n}_{\nu_1\ldots\nu_n} = O^{\mu_1\ldots\mu_n}_{\alpha_1\ldots\alpha_n} T^{\alpha_1\ldots\alpha_n}_{\beta_1\ldots\beta_n} O^{\beta_1\ldots\beta_n}_{\nu_1\ldots\nu_n}$$

$$T^{\alpha_1\ldots\alpha_n}_{\beta_1\ldots\beta_n} = \frac{n+1}{2n+1}\left(g_{\alpha_1\beta_1} - \frac{n}{n+1}\sigma_{\alpha_1\beta_1}\right)\prod_{i=2}^{n} g_{\alpha_i\beta_i}$$

**where**

$$\sigma_{\alpha_i\alpha_j} = \frac{1}{2}(\gamma_{\alpha_i}\gamma_{\alpha_j} - \gamma_{\alpha_j}\gamma_{\alpha_i})$$

**For particle with spin $3/2$ it has form:**

$$P^{\mu}_{\nu} = \frac{1}{2}\left(g^{\perp}_{\mu\nu} - \gamma^{\perp}_{\mu}\gamma^{\perp}_{\nu}/3\right)$$

# $\pi N$ interaction

**States with $J = L - 1/2$ are called '-' states ($1/2^+$, $3/2^-$,$5/2^+$,...) and states with $J = L + 1/2$ are called '+' states ($1/2^-$, $3/2^+$,$5/2^-$,...).**

$$\tilde{N}^+_{\mu_1\ldots\mu_n} = X^{(n)}_{\mu_1\ldots\mu_n} \qquad \tilde{N}^-_{\mu_1\ldots\mu_n} = i\gamma_\nu\gamma_5 X^{(n+1)}_{\nu\mu_1\ldots\mu_n}$$

$$A = \bar{u}(k_1) N^{\pm}_{\mu_1\ldots\mu_L} F^{\mu_1\ldots\mu_{L-1}}_{\nu_1\ldots\nu_{L-1}} N^{\pm}_{\nu_1\ldots\nu_L} u(q_1) BW^{\pm}_L(s) \xrightarrow[c.m.s.]{} \omega^* \left[G(s,t) + H(s,t) i(\vec{\sigma}\vec{n})\right] \omega'$$

$$G(s,t) = \sum_L \left[(L+1)F^+_L(s) - LF^-_L(s)\right] P_L(z) \; ,$$

$$H(s,t) = \sum_L \left[F^+_L(s) + F^-_L(s)\right] P'_L(z) \; .$$

$$F^+_L \quad = (-1)^{L+1}(|\vec{k}||\vec{q}|)^L \sqrt{\chi_i\chi_f}\, \frac{\alpha(L)}{2L+1} BW^+_L(s) \; ,$$

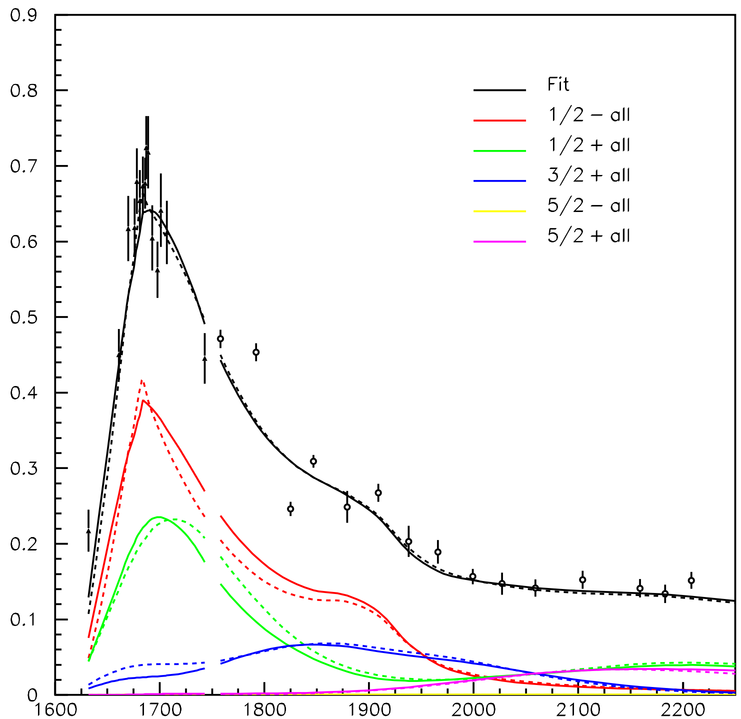
$$F^-_L \quad = (-1)^{L}(|\vec{k}||\vec{q}|)^L \sqrt{\chi_i\chi_f}\, \frac{\alpha(L)}{L} BW^-_L(s) \; .$$

$$\chi_i = m_i + k_{i0} \qquad\qquad \alpha(L) = \prod_{l=1}^{L} \frac{2l-1}{l} = \frac{(2L-1)!!}{L!} \; .$$

# The fit of the the $\pi^- p \to K\Lambda$ reaction

**Full experiment for $\pi N \to K\Lambda$: differential cross section, analyzing power, rotation parameter.**

**A clear evidence for resonances which are hardly seen (or not seen) in the elastic reactions: $N(1710)P_{11}$, $N(1900)P_{13}$,**




**The total cross section for the reaction $\pi^- p \to K^0\Lambda$ and contributions from leading partial waves.**

**Amplitude for the $\pi N$ transition into channels $\pi N$, $\eta N$, $K\Lambda$ and $K\Sigma$:**

$$A_{\pi N} = \omega^* \left[ G(s,t) + H(s,t) i(\vec{\sigma}\vec{n}) \right] \omega' \qquad \vec{n}_j = \varepsilon_{\mu\nu j} \frac{q_\mu k_\nu}{|\vec{k}||\vec{q}|} .$$

$$G(s,t) = \sum_L \left[ (L+1) F_L^+(s) + L F_L^-(s) \right] P_L(z) ,$$

$$H(s,t) = \sum_L \left[ F_L^+(s) - F_L^-(s) \right] P_L'(z) .$$

$z = \cos\Theta$**, the angle of the final meson in c.m.s.**

$$|A|^2 = \frac{1}{2} \mathrm{Tr}\left[ A^*_{\pi N} A_{\pi N} \right] = |G(s,t)|^2 + |H(s,t)|^2 (1 - z^2)$$

**and the recoil asymmetry can be calculated as:**

$$P = \frac{\mathrm{Tr}\left[ A^*_{\pi N} \sigma_2 A_{\pi N} \right]}{2|A|^2 \cos\phi} = \sin\Theta \frac{2 Im\left( H^*(s,t) G(s,t) \right)}{|A|^2} .$$

**Near threshold, only contributions from $S$ and $P$-waves are expected. For the $S_{2I,2J}$ and $P_{2I,2J}$ amplitudes we have**

$$\underline{S_{2I,1}}; \quad G = F_0^+; \quad H = 0; \quad |A|^2 = |F_0^+|^2 \tag{1}$$

$$\underline{P_{2I,1}}; \quad G = F_1^- z; \quad H = -F_1^-; \quad |A|^2 = |F_1^-|^2 \tag{2}$$

$$\underline{P_{2I,3}}; \quad G = 2F_1^+ z; \quad H = F_1^+; \quad |A|^2 = |F_1^+|^2(3z^2+1)$$

**where the indices $(2I, 2J)$ remind of the isospin $I$ and the spin $J$ of the partial waves.**

**The recoil asymmetry vanishes unless different amplitudes interfere.**

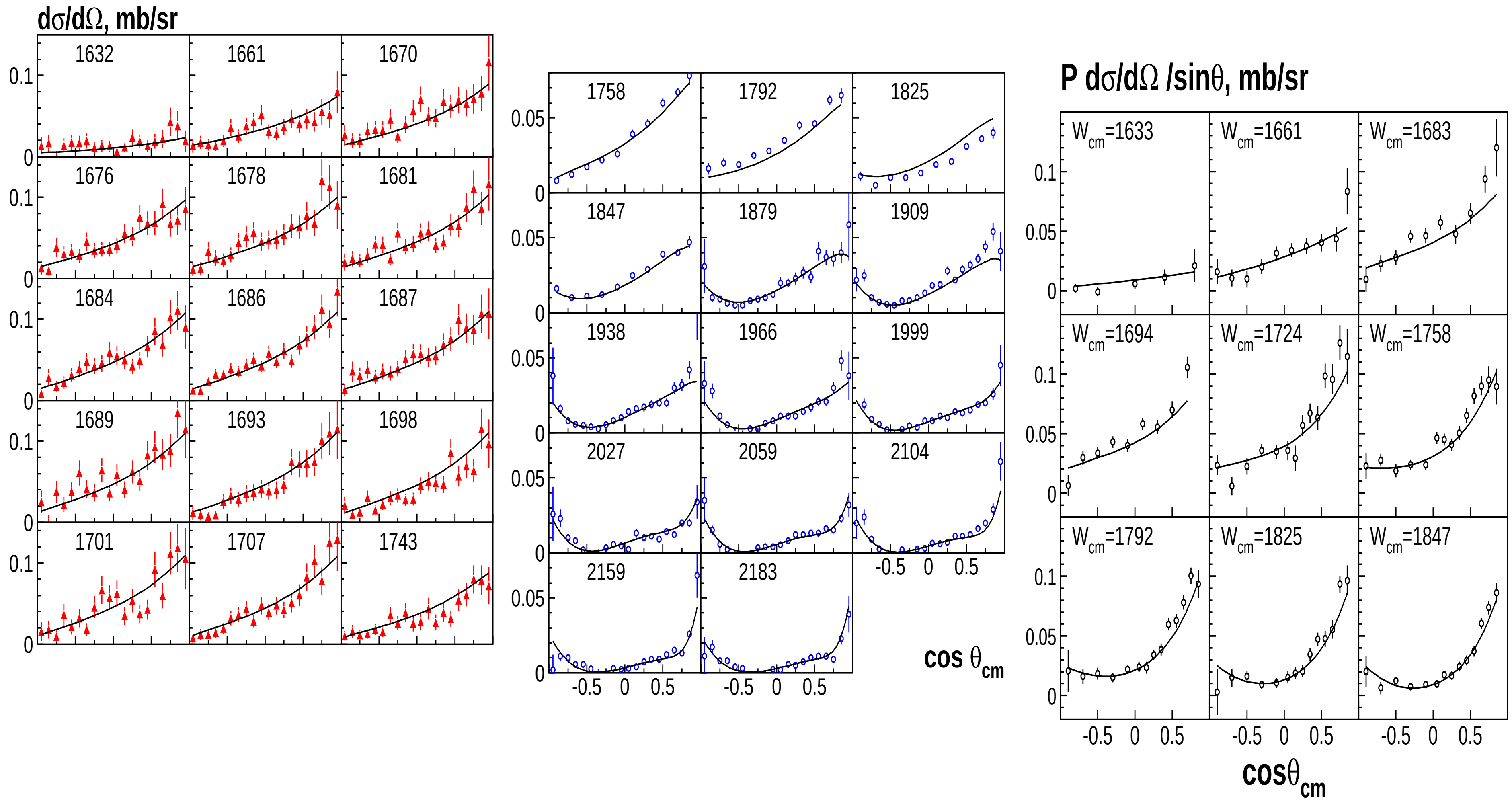
$$\underline{S_{2I,1}+P_{2I,1}} : \quad P\frac{|A|^2}{\sin\Theta} = -2Im(F_0^+ F_1^{-*}) \quad |A|^2 = |F_0^+|^2 + |F_1^-|^2 + 2zRe(F_0^{+*} F_1^-)$$

$$\underline{S_{2I,1}+P_{2I,3}} : \quad P\frac{|A|^2}{\sin\Theta} = 2Im(F_0^+ F_1^{+*}) \quad |A|^2 = |F_0^+|^2 + |F_1^+|^2(3z^2+1) + 4zRe(F_0^{+*} F_1^+)$$

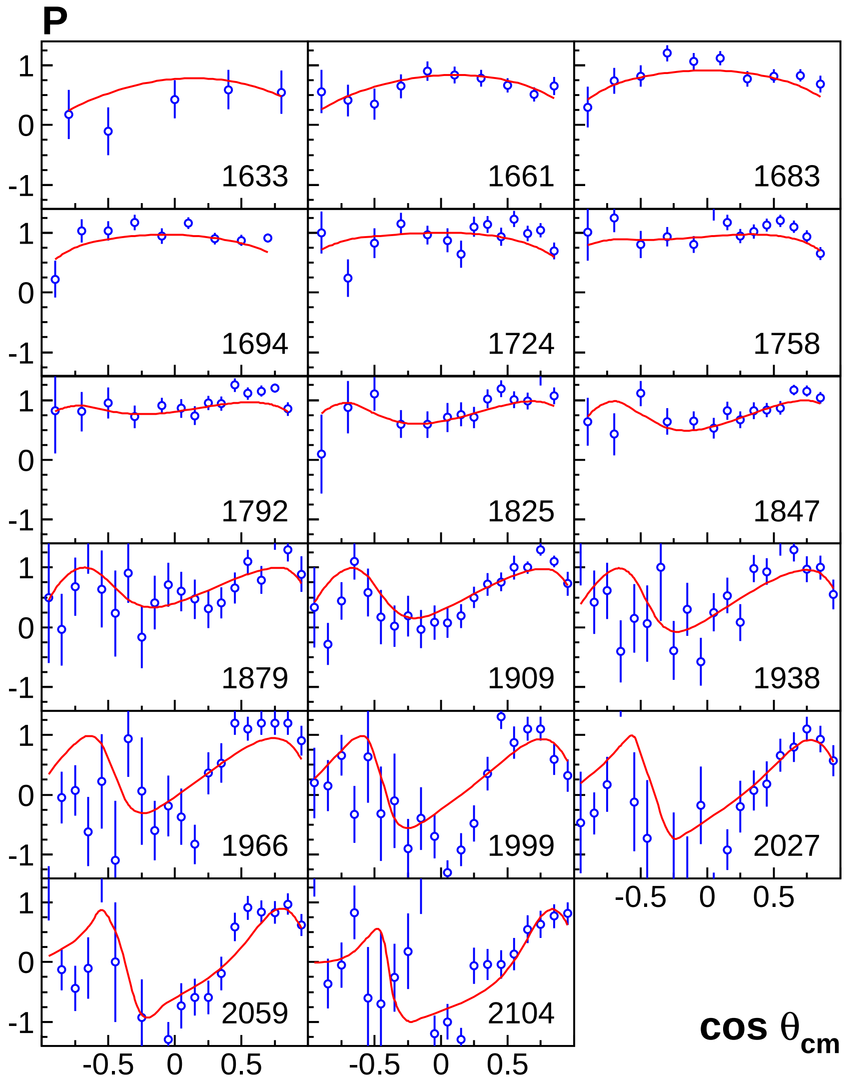
$$\underline{P_{2I,1}+P_{2I,3}} : \quad P\frac{|A|^2}{\sin\Theta} = 6zIm(F_1^{+*} F_1^-) \quad |A|^2 = |F_1^+ - F_1^-|^2 + z^2\left(3|F_1^+|^2 - 2Re(F_1^{+*} F_1^-)\right).$$

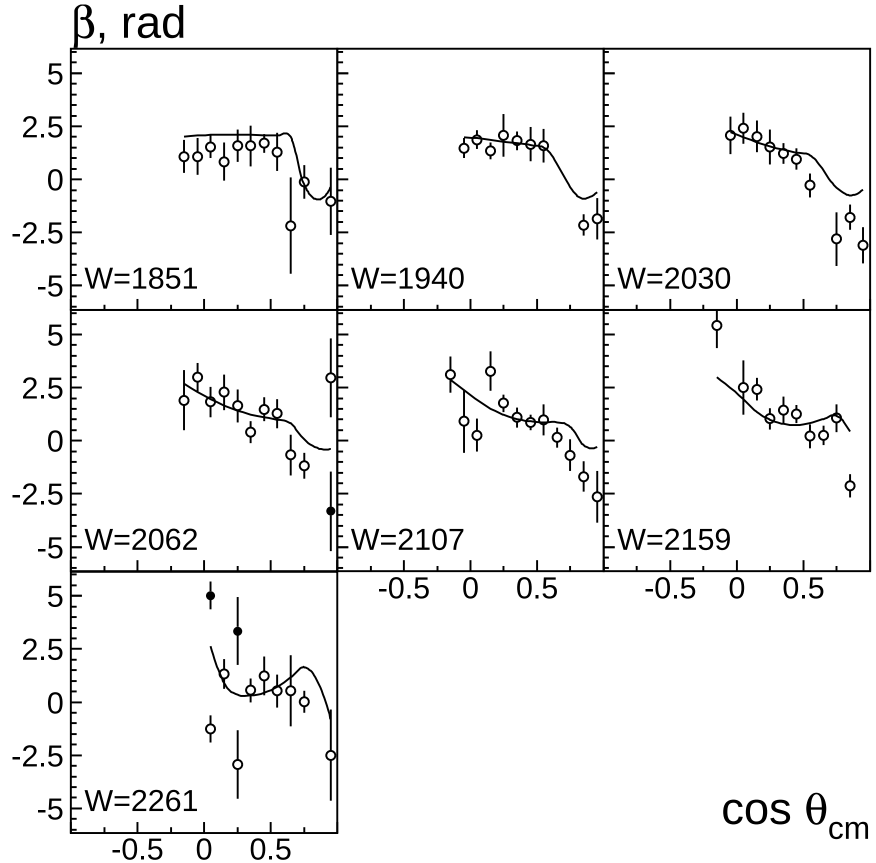
**where $|A|^2$ represents the angular distribution and $P\,|A|^2/\sin\Theta$ an observable proportional to the recoil polarization parameter $P$.**

# The fit of the the $\pi^- p \to K\Lambda$ reaction (differential cross section)

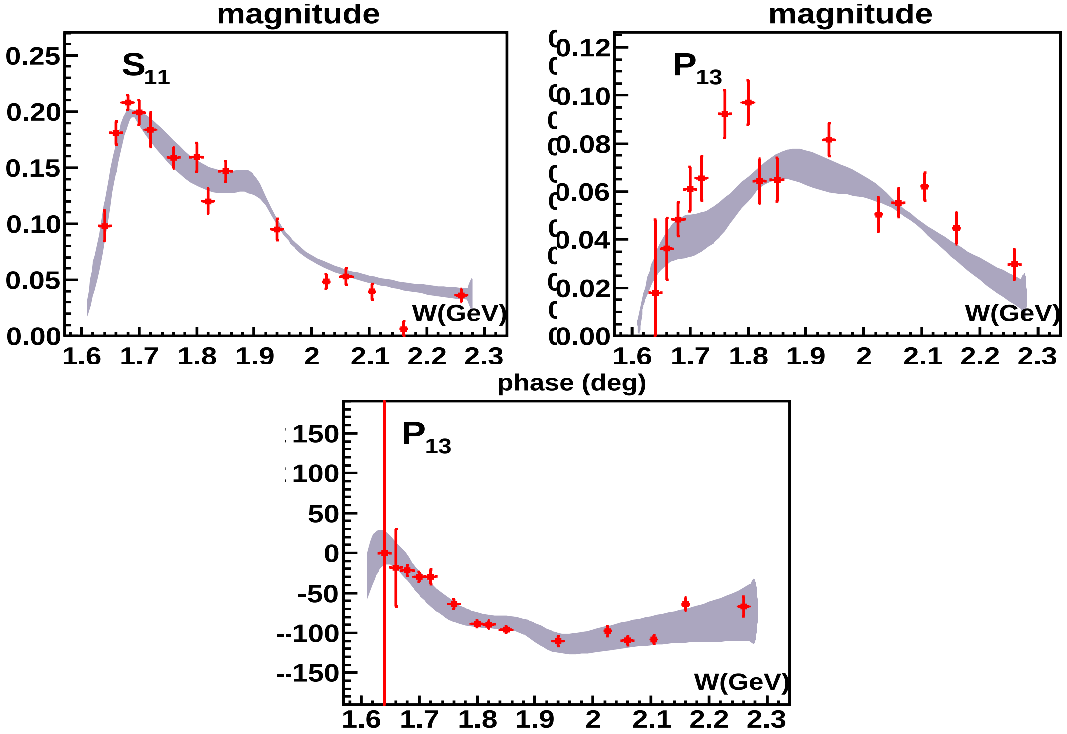


# The fit of the the $\pi^- p \to K\Lambda$ reaction

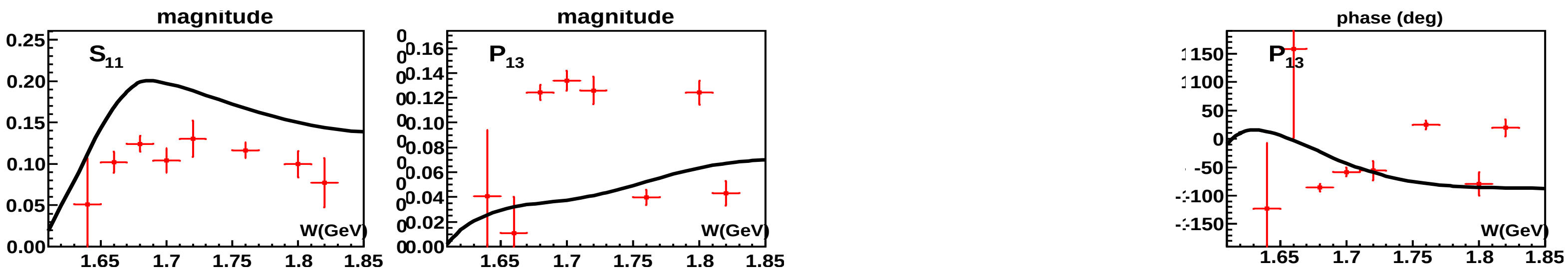


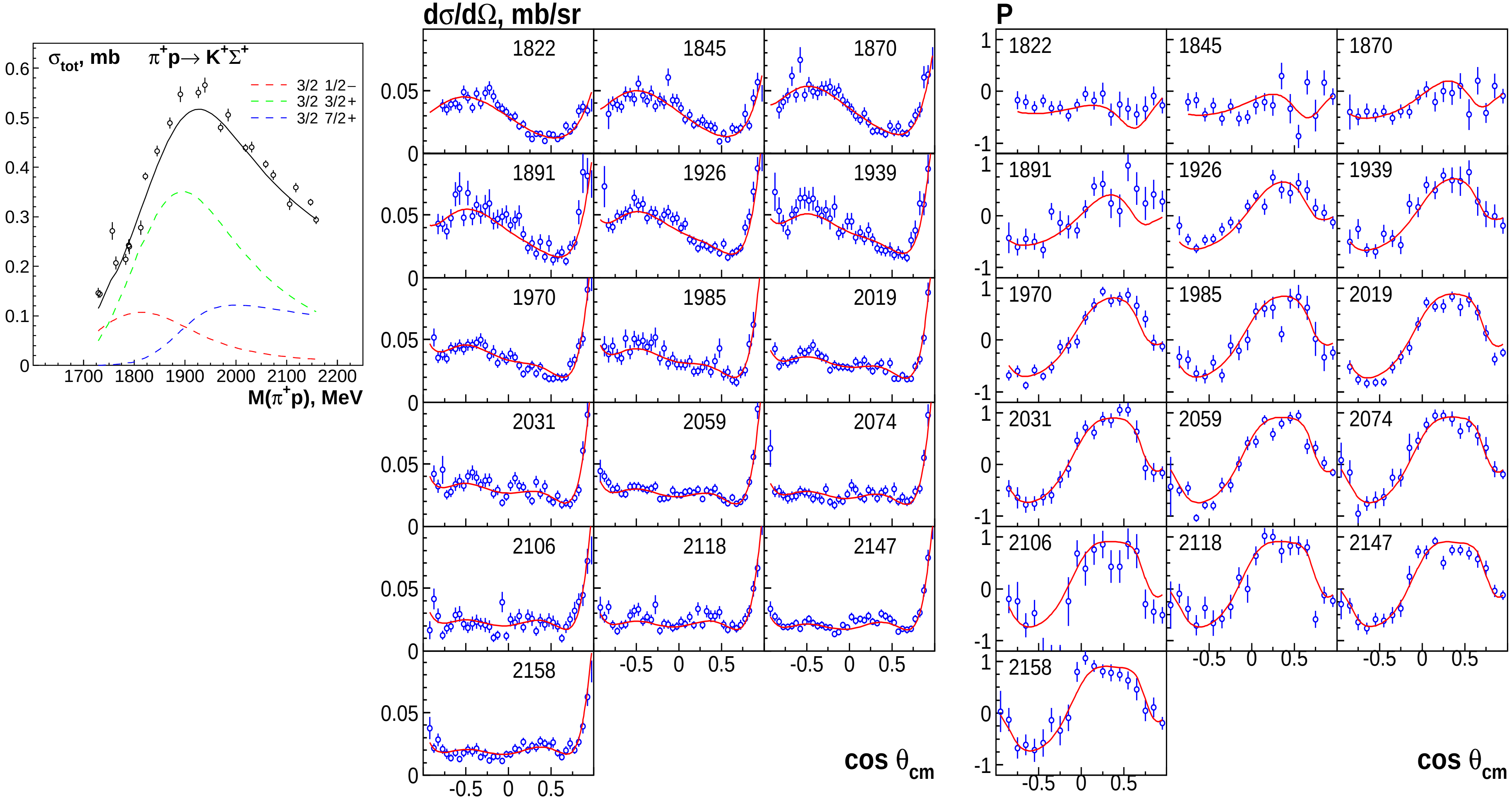
# Energy independent analysis of the $\pi^- p \to K\Lambda$ data




## However this is not a unique solution



# The fit of the the $\pi^+ p \to K^+\Sigma^+$ reaction with BG2011-02



# Status is confirmed. The new properties are defined

Citation: J. Beringer et al. (Particle Data Group), PR D86, 010001 (2012) and 2013 partial update for the 2014 edition (URL: http://pdg.lbl.gov)

## $N(1710)\ 1/2^+$

$I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$ Status: ∗∗∗

The latest GWU analysis (ARNDT 06) finds no evidence for this resonance.

### N(1710) BREIT-WIGNER MASS

| VALUE (MeV) | DOCUMENT ID | | TECN | COMMENT |
|---|---|---|---|---|
| **1680 to 1740 (≈ 1710) OUR ESTIMATE** | | | | |
| 1710±20 | ANISOVICH | 12A | DPWA | Multichannel |
| 1700±50 | CUTKOSKY | 80 | IPWA | $\pi N \to \pi N$ |
| 1723± 9 | HOEHLER | 79 | IPWA | $\pi N \to \pi N$ |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | |
| 1662± 7 | SHRESTHA | 12A | DPWA | Multichannel |
| 1725±25 | ANISOVICH | 10 | DPWA | Multichannel |
| 1729±16 | [1] BATINIC | 10 | DPWA | $\pi N \to N\pi, N\eta$ |
| 1752± 3 | PENNER | 02C | DPWA | Multichannel |
| 1699±65 | VRANA | 00 | DPWA | Multichannel |
| 1720±10 | ARNDT | 96 | IPWA | $\gamma N \to \pi N$ |
| 1717±28 | MANLEY | 92 | IPWA | $\pi N \to \pi N$ & $N\pi\pi$ |
| 1706 | CUTKOSKY | 90 | IPWA | $\pi N \to \pi N$ |
| 1730 | SAXON | 80 | DPWA | $\pi^- p \to \Lambda K^0$ |
| 1720 | [2] LONGACRE | 77 | IPWA | $\pi N \to N\pi\pi$ |
| 1710 | [3] LONGACRE | 75 | IPWA | $\pi N \to N\pi\pi$ |

### N(1710) BREIT-WIGNER WIDTH

| VALUE (MeV) | DOCUMENT ID | | TECN | COMMENT |
|---|---|---|---|---|
| **50 to 250 (≈ 100) OUR ESTIMATE** | | | | |
| 200± 18 | ANISOVICH | 12A | DPWA | Multichannel |
| 93± 30 | CUTKOSKY | 90 | IPWA | $\pi N \to \pi N$ |
| 90± 30 | CUTKOSKY | 80 | IPWA | $\pi N \to \pi N$ |
| 120± 15 | HOEHLER | 79 | IPWA | $\pi N \to \pi N$ |

Citation: J. Beringer et al. (Particle Data Group), PR D86, 010001 (2012) and 2013 partial update for the 2014 edition (URL: http://pdg.lbl.gov)

## $\Delta(1920)\ 3/2^+$

$I(J^P) = \frac{3}{2}(\frac{3}{2}^+)$ Status: ∗∗∗

The latest GWU analysis (ARNDT 06) finds no evidence for this resonance.

### Δ(1920) BREIT-WIGNER MASS

| VALUE (MeV) | DOCUMENT ID | | TECN | COMMENT |
|---|---|---|---|---|
| **1900 to 1970 (≈ 1920) OUR ESTIMATE** | | | | |
| 1900 ± 30 | ANISOVICH | 12A | DPWA | Multichannel |
| 1920 ± 80 | CUTKOSKY | 80 | IPWA | $\pi N \to \pi N$ |
| 1868 ± 10 | HOEHLER | 79 | IPWA | $\pi N \to \pi N$ |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | |
| 2146 ± 32 | SHRESTHA | 12A | DPWA | Multichannel |
| 1990 ± 35 | HORN | 08A | DPWA | Multichannel |
| 2057 ± 1 | PENNER | 02C | DPWA | Multichannel |
| 1889 ±100 | VRANA | 00 | DPWA | Multichannel |
| 2014 ± 16 | MANLEY | 92 | IPWA | $\pi N \to \pi N$ & $N\pi\pi$ |
| 1840 ± 40 | CANDLIN | 84 | DPWA | $\pi^+ p \to \Sigma^+ K^+$ |
| 1955.0± 13.0 | [1] CHEW | 80 | BPWA | $\pi^+ p \to \pi^+ p$ |
| $2065.0^{+13.6}_{-12.9}$ | [1] CHEW | 80 | BPWA | $\pi^+ p \to \pi^+ p$ |

### Δ(1920) BREIT-WIGNER WIDTH

| VALUE (MeV) | DOCUMENT ID | | TECN | COMMENT |
|---|---|---|---|---|
| **180 to 300 (≈ 260) OUR ESTIMATE** | | | | |
| 310 ± 60 | ANISOVICH | 12A | DPWA | Multichannel |
| 300 ±100 | CUTKOSKY | 80 | IPWA | $\pi N \to \pi N$ |
| 220 ± 80 | HOEHLER | 79 | IPWA | $\pi N \to \pi N$ |

# Gauge invariant $\gamma N$ vertices

**Photon has quantum numbers $J^{PC}=1^{--}$, proton $1/2^+$. Then in S-wave two states can be formed is $1/2^-$ and $3/2^-$. Then P-wave $1/2^+, 3/2^+$ and $1/2^+, 3/2^+, 5/2^+$.**

$$V^{(1+)\mu}_{\alpha_1\ldots\alpha_n} = \gamma^{\perp\perp}_\mu i\gamma_5 X^{(n)}_{\alpha_1\ldots\alpha_n}\ , \qquad V^{(1-)\mu}_{\alpha_1\ldots\alpha_n} = \gamma_\xi \gamma^{\perp\perp}_\mu X^{(n+1)}_{\xi\alpha_1\ldots\alpha_n}\ ,$$
$$V^{(2+)\mu}_{\alpha_1\ldots\alpha_n} = \gamma_\nu i\gamma_5 X^{(n+1)}_{\nu\alpha_1\ldots\alpha_n} g^{\perp\perp}_{\mu\alpha_n}\ , \quad V^{(2-)\mu}_{\alpha_1\ldots\alpha_n} = X^{(n-1)}_{\alpha_2\ldots\alpha_n} g^{\perp\perp}_{\alpha_1\mu}$$
$$V^{(3+)\mu}_{\alpha_1\ldots\alpha_n} = \hat{k} i\gamma_5 X^{(n)}_{\alpha_1\ldots\alpha_n} Z_\mu\ , \qquad V^{(3-)\mu}_{\alpha_1\ldots\alpha_n} = \hat{k}\gamma_\chi X^{(n+1)}_{\chi\alpha_1\ldots\alpha_n} Z_\mu\ ,\ .$$

$$Z_\mu = \left((Pk^\gamma)k^\gamma_\mu - (k^\gamma)^2 P_\mu\right)$$

$$\gamma^{\perp\perp}_\mu = \gamma_\nu g^{\perp\perp}_{\mu\nu} \qquad\qquad g^{\perp\perp}_{\nu\mu} = \left(g_{\nu\mu} - \frac{P_\nu P_\nu}{P^2} - \frac{k^\perp_\nu k^\perp_\nu}{k^2_\perp}\right)$$

**General structure of the single–meson electro-production amplitude:**

$$J_\mu = i\mathcal{F}_1\tilde{\sigma}_\mu + \mathcal{F}_2(\vec{\sigma}\vec{q})\frac{\varepsilon_{\mu ij}\sigma_i k_j}{|\vec{k}||\vec{q}|} + i\mathcal{F}_3\frac{(\vec{\sigma}\vec{k})}{|\vec{k}||\vec{q}|}\tilde{q}_\mu + i\mathcal{F}_4\frac{(\vec{\sigma}\vec{q})}{\vec{q}^2}\tilde{q}_\mu$$

$$+ i\mathcal{F}_5\frac{(\vec{\sigma}\vec{k})}{|\vec{k}|^2}k_\mu + i\mathcal{F}_6\frac{(\vec{\sigma}\vec{q})}{|\vec{q}||\vec{k}|}k_\mu \qquad \mu = 1, 2, 3,$$

$$\mathcal{F}_1(z) = \sum_{L=0}^{\infty} [LM_L^+ + E_L^+]P'_{L+1}(z) + [(L+1)M_L^- + E_L^-]P'_{L-1}(z),$$

$$\mathcal{F}_2(z) = \sum_{L=1}^{\infty} [(L+1)M_L^+ + LM_L^-]P'_L(z)\ ,$$

$$\mathcal{F}_3(z) = \sum_{L=1}^{\infty} [E_L^+ - M_L^+]P''_{L+1}(z) + [E_L^- + M_L^-]P''_{L-1}(z)\ ,$$

$$\mathcal{F}_4(z) = \sum_{L=2}^{\infty} [M_L^+ - E_L^+ - M_L^- - E_L^-]P''_L(z)\ ,$$

$$\mathcal{F}_5(z) = \sum_{L=0}^{\infty} [(L+1)S_L^+ P'_{L+1}(z) - LS_L^- P'_{L-1}(z)]\ ,$$

$$\mathcal{F}_6(z) = \sum_{L=1}^{\infty} [LS_L^- - (L+1)S_L^+]P'_L(z)$$

**For the positive states $J = L + 1/2 \quad (L{=}n)$:**

$$J^{i+}_{\mu} \quad = \quad \varepsilon_\mu \bar{u}(q_N) X^{(n)}_{\alpha_1\ldots\alpha_n}(q^\perp) F^{\alpha_1\ldots\alpha_n}_{\beta_1\ldots\beta_n} V^{(i+)\mu}_{\beta_1\ldots\beta_n}(k^\perp) u(k_N)$$

$$\begin{array}{lll}
\mathcal{F}^{1+}_1 = \lambda_n P'_{n+1} & \mathcal{F}^{2+}_1 = 0 & \mathcal{F}^{3+}_1 = 0 \\
\mathcal{F}^{1+}_2 = \lambda_n P'_{n} & \mathcal{F}^{2+}_2 = -\frac{\lambda_n}{n} P'_{n} & \mathcal{F}^{3+}_2 = 0 \\
\mathcal{F}^{1+}_3 = 0 & \mathcal{F}^{2+}_3 = \frac{\lambda_n}{n} P''_{n+1} & \mathcal{F}^{3+}_3 = 0 \\
\mathcal{F}^{1+}_4 = 0 & \mathcal{F}^{2+}_4 = \frac{\lambda_n}{n} P''_{n} & \mathcal{F}^{3+}_4 = 0 \\
\mathcal{F}^{1+}_5 = 0 & \mathcal{F}^{2+}_5 = 0 & \mathcal{F}^{3+}_5 = +\xi_n P'_{n+1} \\
\mathcal{F}^{1+}_6 = 0 & \mathcal{F}^{2+}_6 = 0 & \mathcal{F}^{3+}_6 = -\xi_n P'_{n}
\end{array}$$

$$\lambda_n = \frac{\alpha_n}{2n+1}(|\vec{k}||\vec{q}|)^n \chi_i \chi_f \qquad \chi_{i,f} = \sqrt{m_{i,f} + k_{0i,f}}$$

$$\xi_n = (k^\gamma)^2 \frac{\alpha_n}{2n+1}(|\vec{k}||\vec{q}|)^n \chi_i \chi_f$$

**No singularities and the correct behavior at $Q^2 \to 0$.**

# Meson Photoproduction experiments

- **GRAAL (Grenoble): Polarized beam. Ideal for the beam asymmetry and double polarization observables for hyperon final states.**
  $\gamma p \to \pi^0 p, \eta p, K\Lambda, K\Sigma, \pi^0\pi^0 p, \pi^0\eta p, \omega p$, $\gamma n \to \pi^0 n, \eta n$.

- **CLAS (JLAB): High statistic, very good detector of charged particles:**
  $\gamma p \to \pi^- n, K\Lambda, K\Sigma, \pi^+\pi^- p, \omega p$. **As missing mass data** $\gamma p \to \pi^0 p, \eta p$.
  **Data on deuterium target. Energy is up to W=2.5 GeV.**
  **Analysis: EBAC, SAID and recently Bonn-Gatchina.**

- **MAMI (Mainz): High statistic, very good detector of neutral particles: (Crystal Ball):**
  $\gamma p \to \pi^0 p, K\Lambda, K\Sigma, \pi^0\pi^0 p, \pi^0\eta p$, $\gamma n \to \eta n, \pi^0 n, \pi^0\pi^0 n, \pi^0\eta n$.
  **Energy is only up to W=1.85 GeV. Analysis: MAID and Bonn-Gatchina.**

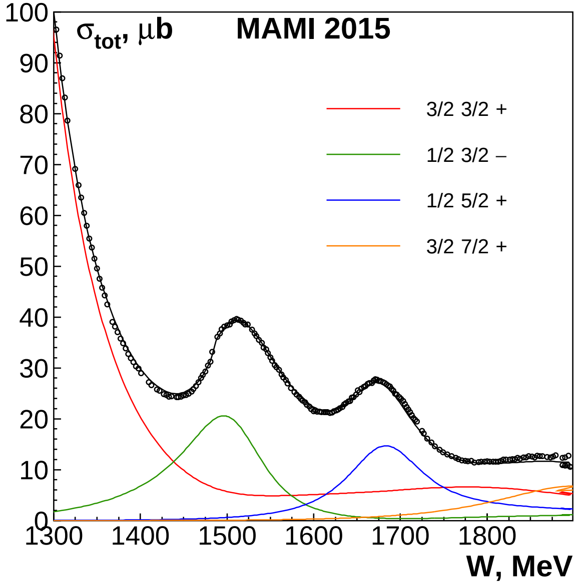
- **CB-ELSA (Bonn): Moderate statistic, very good detector of neutral particles: (Crystal Barrel):** $\gamma p \to \pi^0 p, \eta p, \pi^0\pi^0 p, \pi^0\eta p, \omega p$, $\gamma n \to \eta n, \pi^0 n$. **Energy is up to W=2.3 GeV. Analysis: Bonn-Gatchina.**

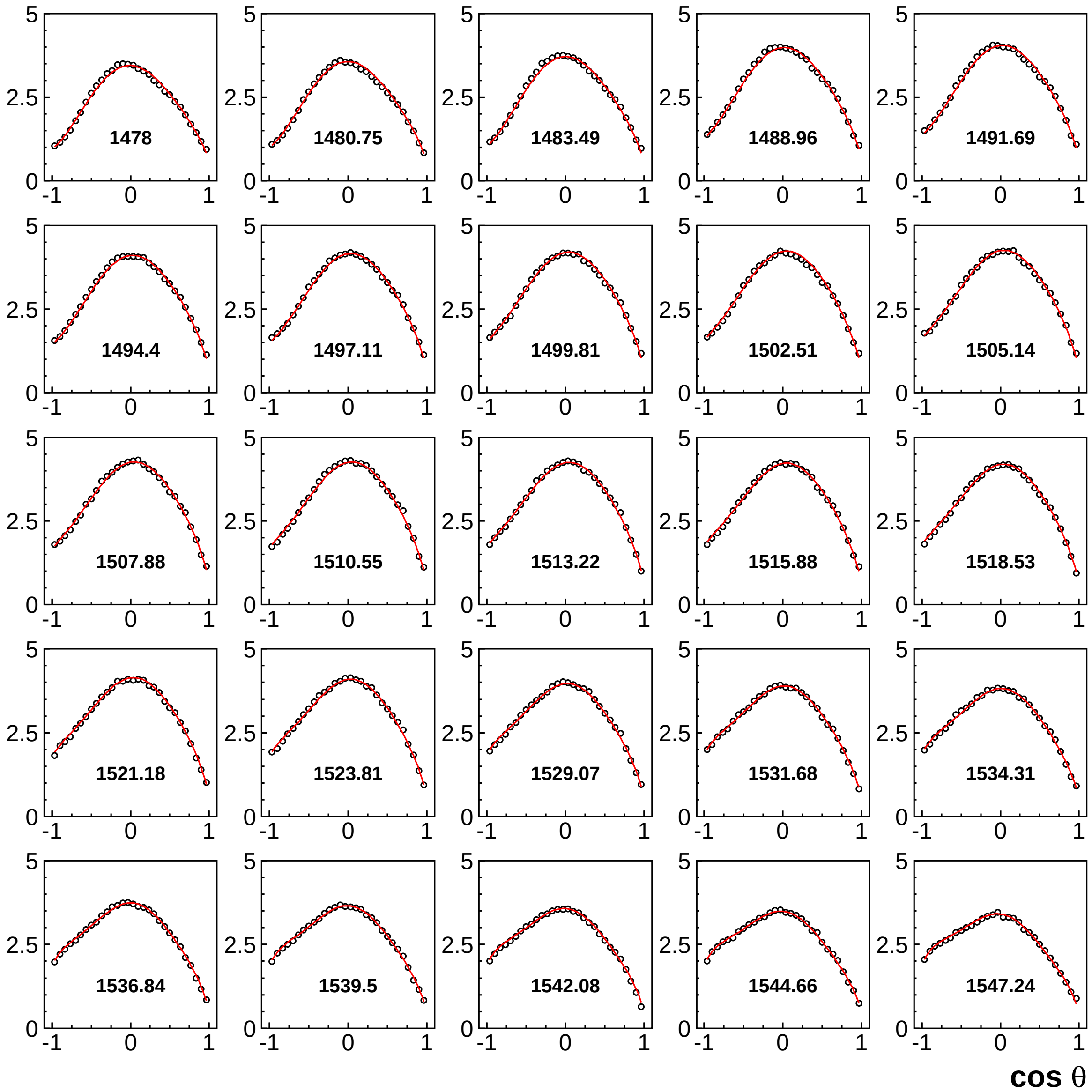
- **Independent analysis groups: Jülich (M.Doering), OSAKA (T. Sato), Giessen (V. Shklyar), M. Manley (Kent Uni)**

- **The main task: search for new baryon resonances**
- **Polarization data are sensitive to weak signals**
- **Double polarization data are available** $C_x, C_z$, $O_x, O_z$, $E, G, H$ **.**
- **Double polarization observables (assuming** $XZ$ **is the reaction plane)**

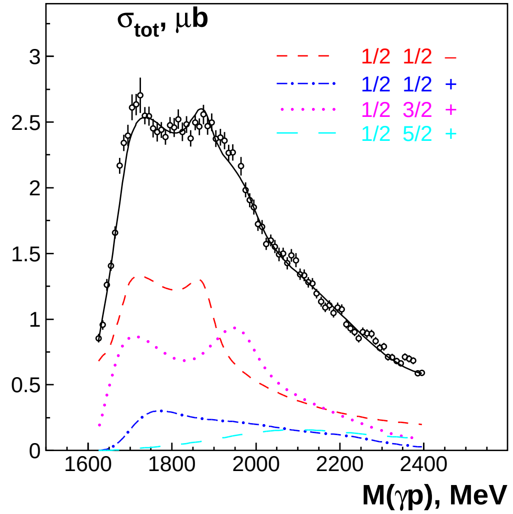
| **Photon** | | **Target** | | | **Recoil** | | | **Target + Recoil** | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | $-$ | $-$ | $-$ | $-$ | $x'$ | $y'$ | $z'$ | $x'$ | $x'$ | $z'$ | $z'$ |
| | $-$ | $x$ | $y$ | $z$ | $-$ | $-$ | $-$ | $x$ | $z$ | $x$ | $z$ |
| **unpol.** | $\sigma_0$ | $0$ | $T$ | $0$ | $0$ | $P$ | $0$ | $T_{x'}$ | $-L_{x'}$ | $T_{z'}$ | $L_{z'}$ |
| **lin.pol.** | $-\Sigma$ | $H$ | $-P$ | $-G$ | $O_{x'}$ | $-T$ | $O_{z'}$ | $-L_{z'}$ | $T_{z'}$ | $-L_{x'}$ | $-T_{x'}$ |
| **circ.pol.** | $0$ | $F$ | $0$ | $-E$ | $-C_{x'}$ | $0$ | $-C_{z'}$ | $0$ | $0$ | $0$ | $0$ |

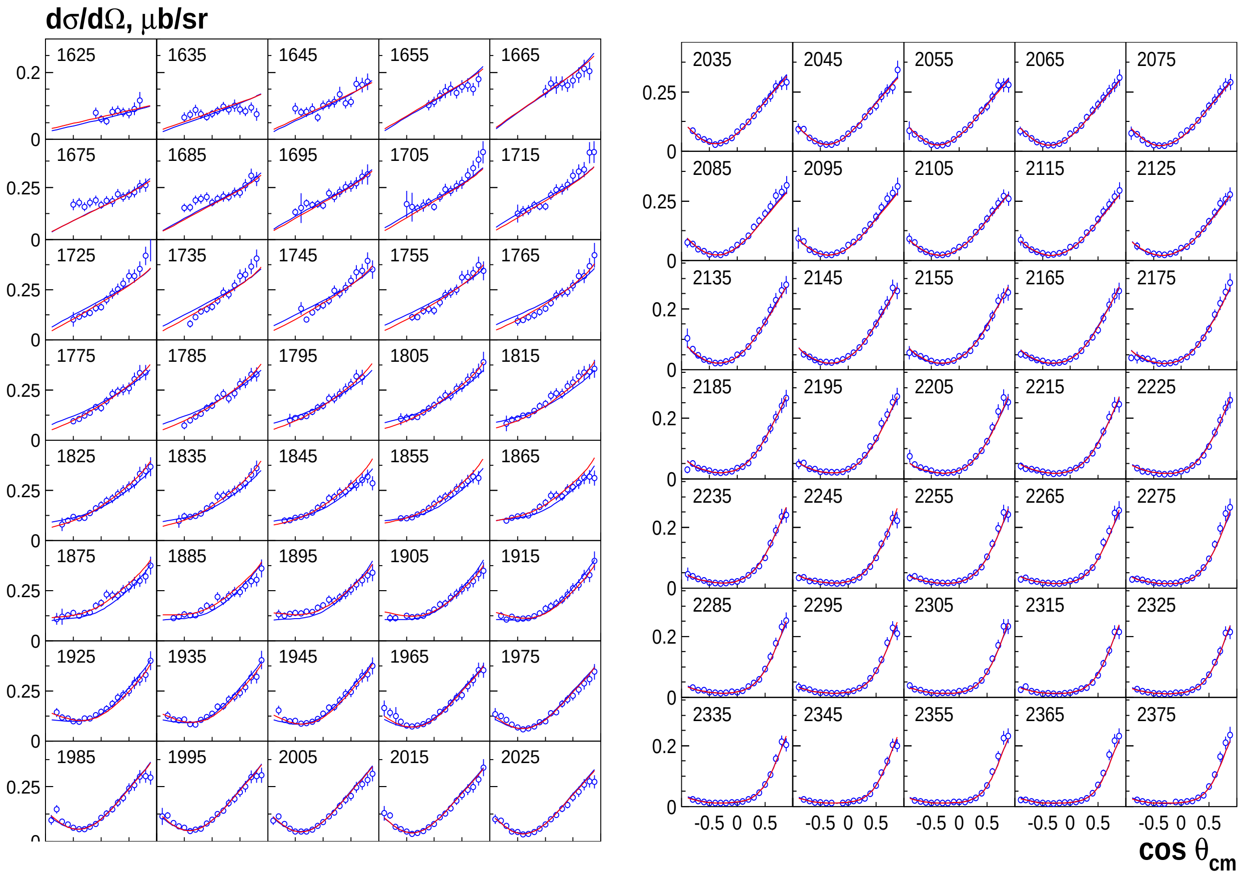
# New MAMI data on $\gamma p \to \pi^0 p$



# The $\gamma p \to K\Lambda$ reaction (CLAS 2009)

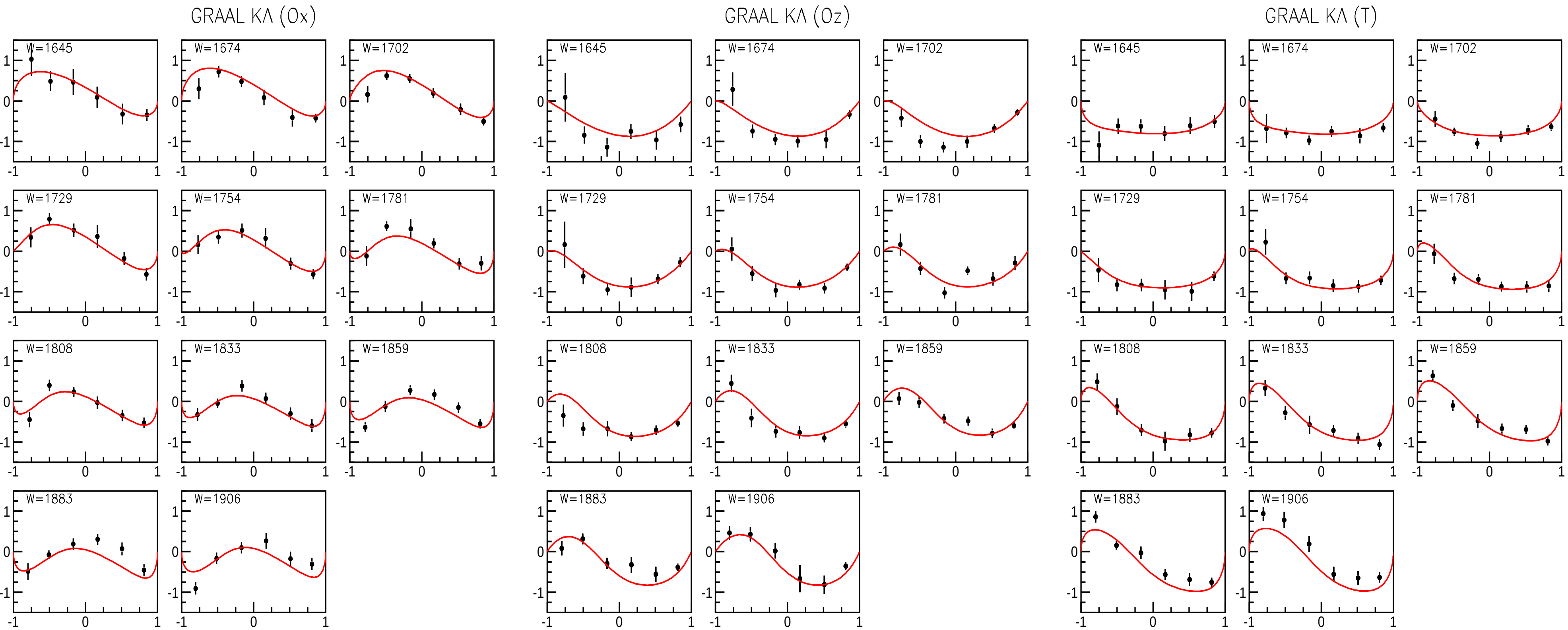



**New $S_{11}$ state with mass $1890 \pm 10$ MeV and width $90 \pm 10$ MeV improves description of the data.**

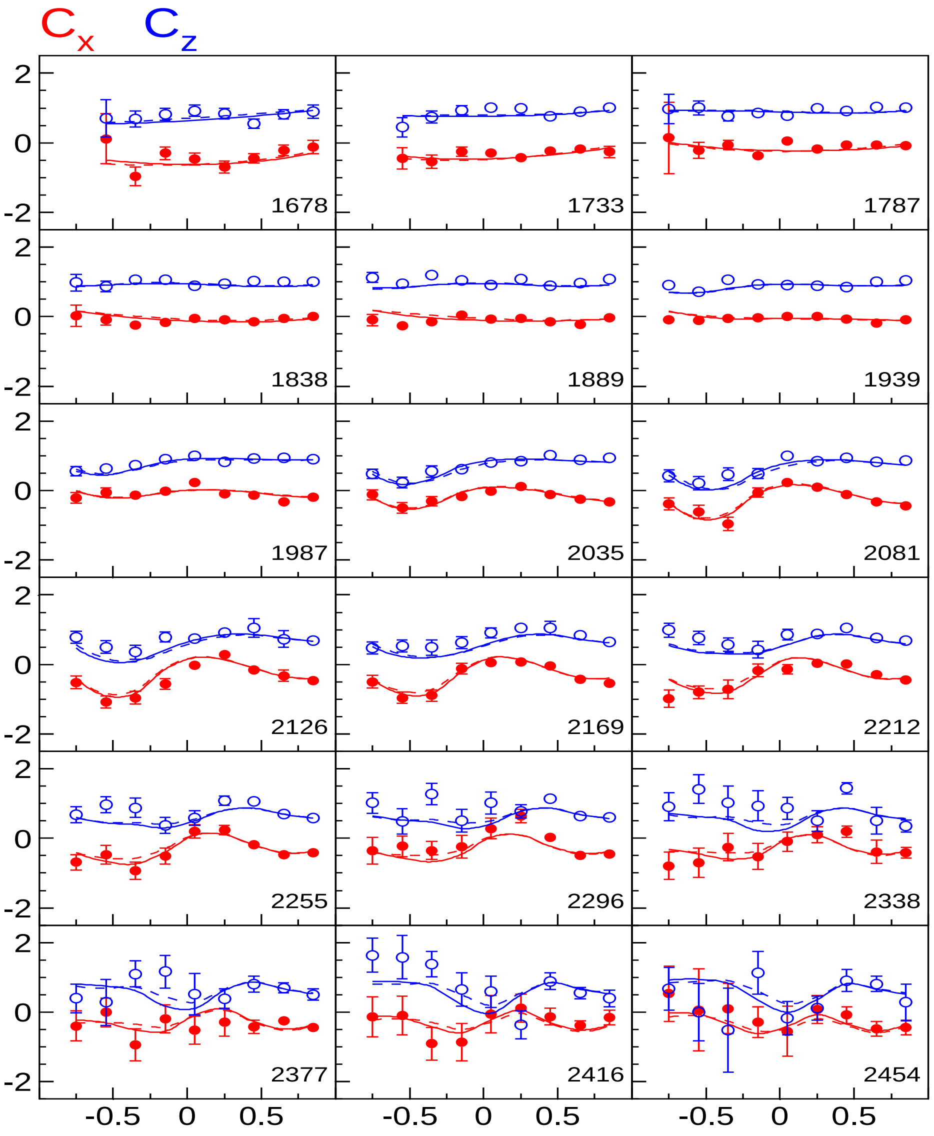
# The $O_x$, $O_z$ and $T$ (GRAAL) observables from $\gamma p \to K\Lambda$

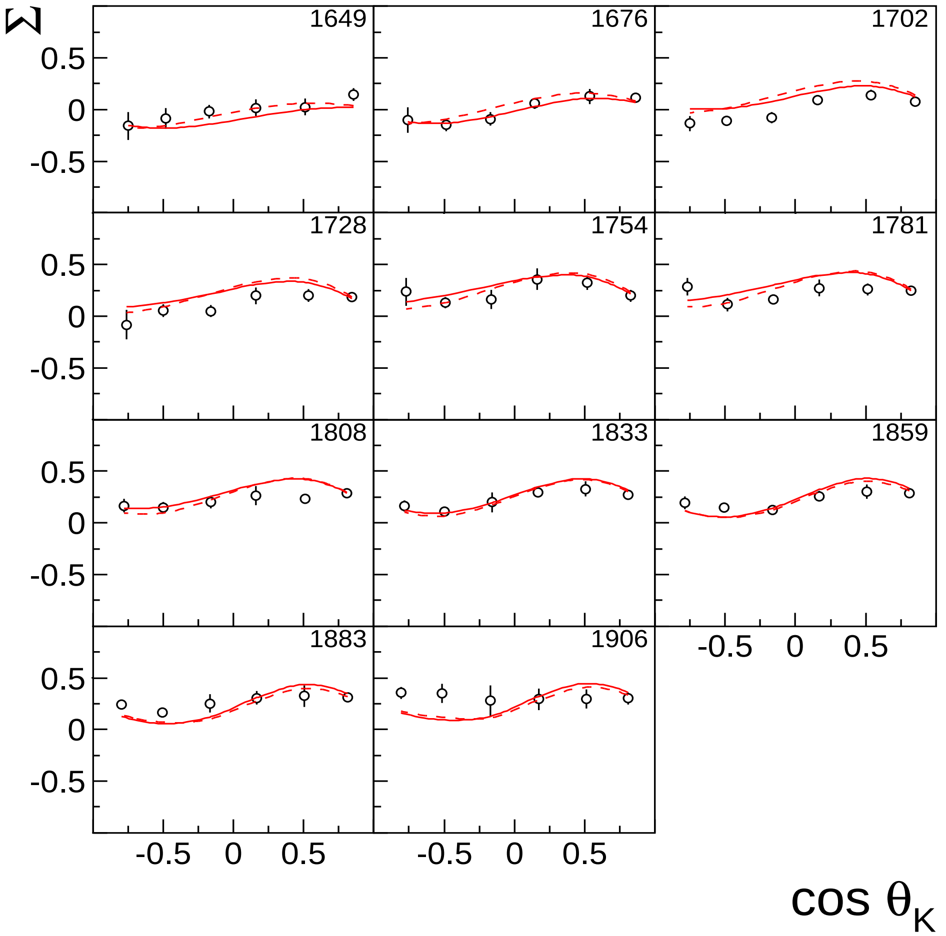
## description is notably improved with $S_{11}(1890)$



# The $\gamma p \to K^+\Lambda$: $C_x$, $C_z$ (CLAS) and beam asymmetry (GRAAL)

## BG2011-02 M (dashed) BG2013-02 (solid)



# New resonances are found. One of them has 3* and was proposed to be defined as 4* state

Citation: J. Beringer *et al.* (Particle Data Group), PR **D86**, 010001 (2012) and 2013 partial update for the 2014 edition (URL: http://pdg.lbl.gov)

## $N(1895)\ 1/2^-$

$I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$ Status: $**$

OMITTED FROM SUMMARY TABLE

The latest GWU analysis (ARNDT 06) finds no evidence for this resonance.

### N(1895) BREIT-WIGNER MASS

| *VALUE* (MeV) | *DOCUMENT ID* | | *TECN* | *COMMENT* |
|---|---|---|---|---|
| **≈ 2090 OUR ESTIMATE** | | | | |
| $1895 \pm 15$ | ANISOVICH | 12A | DPWA | Multichannel |
| $2180 \pm 80$ | CUTKOSKY | 80 | IPWA | $\pi N \to \pi N$ |
| $1880 \pm 20$ | HOEHLER | 79 | IPWA | $\pi N \to \pi N$ |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | |
| $1910 \pm 15$ | SHRESTHA | 12A | DPWA | Multichannel |
| $1812 \pm 25$ | BATINIC | 10 | DPWA | $\pi N \to N\pi$, $N\eta$ |
| $1822 \pm 43$ | VRANA | 00 | DPWA | Multichannel |
| $1897 \pm 50^{+30}_{-2}$ | PLOETZKE | 98 | SPEC | $\gamma p \to p\eta'(958)$ |
| $1928 \pm 59$ | MANLEY | 92 | IPWA | $\pi N \to \pi N$ & $N\pi\pi$ |

### N(1895) BREIT-WIGNER WIDTH

| *VALUE* (MeV) | *DOCUMENT ID* | | *TECN* | *COMMENT* |
|---|---|---|---|---|
| $90^{+30}_{-15}$ | ANISOVICH | 12A | DPWA | Multichannel |
| $350 \pm 100$ | CUTKOSKY | 80 | IPWA | $\pi N \to \pi N$ |
| $95 \pm 30$ | HOEHLER | 79 | IPWA | $\pi N \to \pi N$ |

Citation: J. Beringer *et al.* (Particle Data Group), PR **D86**, 010001 (2012) and 2013 partial update for the 2014 edition (URL: http://pdg.lbl.gov)

## $N(1900)\ 3/2^+$

$I(J^P) = \frac{1}{2}(\frac{3}{2}^+)$ Status: $***$

The latest GWU analysis (ARNDT 06) finds no evidence for this resonance.

### N(1900) BREIT-WIGNER MASS

| *VALUE* (MeV) | *DOCUMENT ID* | | *TECN* | *COMMENT* |
|---|---|---|---|---|
| **≈ 1900 OUR ESTIMATE** | | | | |
| $1905 \pm 30$ | ANISOVICH | 12A | DPWA | Multichannel |
| $1915 \pm 60$ | NIKONOV | 08 | DPWA | Multichannel |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | |
| $1900 \pm 8$ | SHRESTHA | 12A | DPWA | Multichannel |
| $1951 \pm 53$ | PENNER | 02C | DPWA | Multichannel |
| $1879 \pm 17$ | MANLEY | 92 | IPWA | $\pi N \to \pi N$ & $N\pi\pi$ |

### N(1900) BREIT-WIGNER WIDTH

| *VALUE* (MeV) | *DOCUMENT ID* | | *TECN* | *COMMENT* |
|---|---|---|---|---|
| **∼ 250 OUR ESTIMATE** | | | | |
| $250^{+120}_{-50}$ | ANISOVICH | 12A | DPWA | Multichannel |
| $180 \pm 40$ | NIKONOV | 08 | DPWA | Multichannel |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | |
| $101 \pm 15$ | SHRESTHA | 12A | DPWA | Multichannel |
| $622 \pm 42$ | PENNER | 02C | DPWA | Multichannel |
| $498 \pm 78$ | MANLEY | 92 | IPWA | $\pi N \to \pi N$ & $N\pi\pi$ |

# Parameterization of the partial wave amplitude

$$A_{1i} = K_{1j}(I - i\rho K)^{-1}_{ji}$$

**and**

$$K_{ij} = \sum_{\alpha} \frac{g_i^\alpha g_j^\alpha}{M_\alpha^2 - s} + f_{ij}(s) \qquad f_{ij} = \frac{f_{ij}^{(1)} + f_{ij}^{(2)}\sqrt{s}}{s - s_0^{ij}} \,.$$

**where $f_{ij}$ is non-resonant transition part.**

**For the small coupled initial state, e.g. photoproduction:**

$$A_k = P_j(I - i\rho K)^{-1}_{jk}$$

**The vector of the initial interaction has the form:**

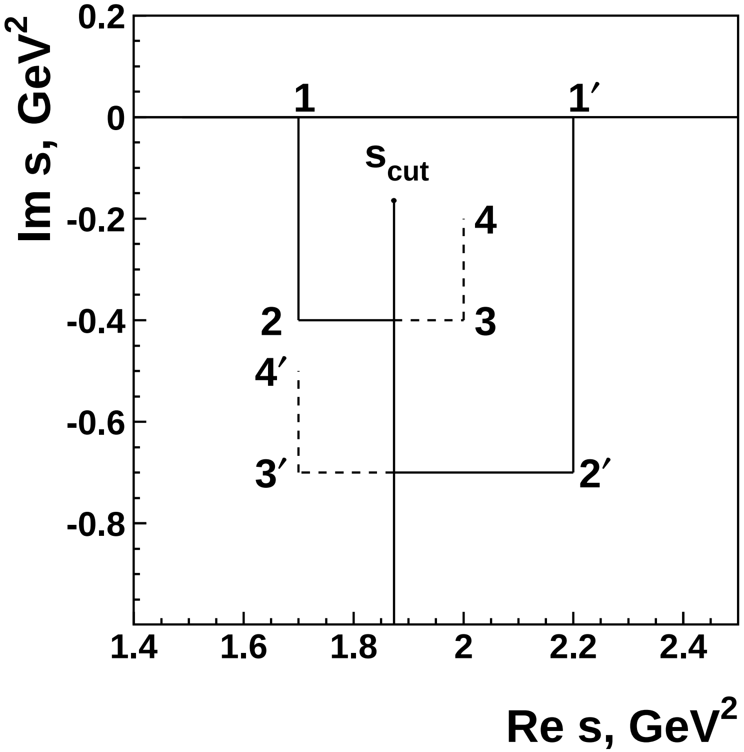
$$P_j = \sum_{\alpha} \frac{\Lambda^\alpha g_j^\alpha}{M_\alpha^2 - s} + F_j(s)$$

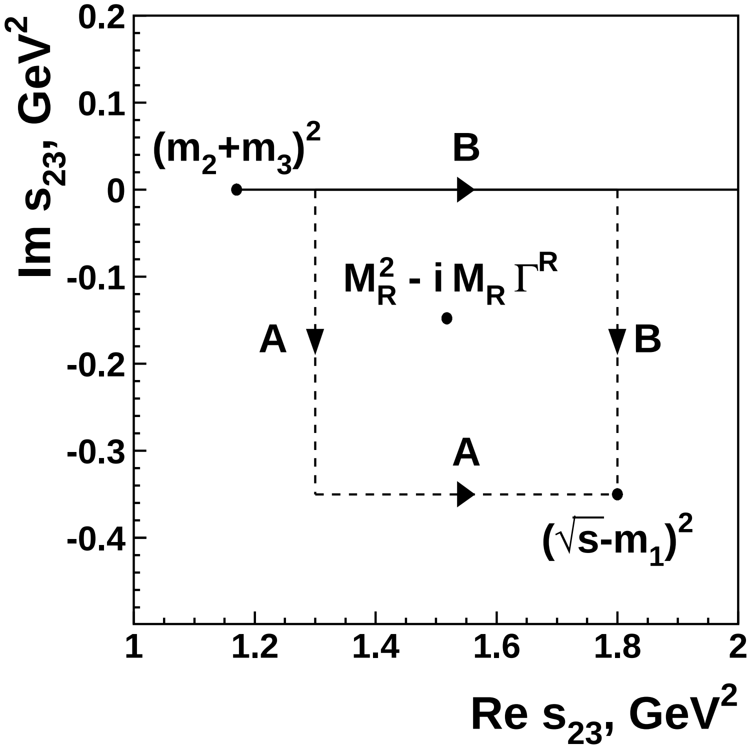
**Here $F_j$ is non-resonant production of the final state $j$.**

## Three body phase volume:

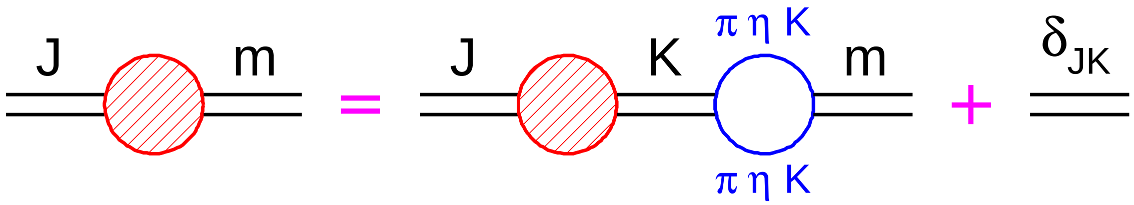
$$\rho_3(s) = \int\limits_{(m_2+m_3)^2}^{(\sqrt{s}-m_1)^2} \frac{ds_{23}}{\pi} \frac{\rho(s,\sqrt{s_{23}},m_1)\, M_R\Gamma^R_{tot}}{(M_R^2-s_{23})^2+(M_R\Gamma^R_{tot})^2} \,,$$

$$M_R\Gamma^R_{tot} = \rho(s_{23},m_2,m_3)g^2(s_{23}) \,,$$



# N/D based (D-matrix) analysis of the data




$$D_{jm} = D_{jk} \sum_\alpha B_\alpha^{km}(s) \frac{1}{M_m - s} + \frac{\delta_{jm}}{M_j^2 - s} \qquad \hat{D} = \hat{\kappa}(I - \hat{B}\hat{\kappa})^{-1}$$

$$\hat{\kappa} = diag\left(\frac{1}{M_1^2 - s}, \frac{1}{M_2^2 - s}, \ldots, \frac{1}{M_N^2 - s}, R_1, R_2 \ldots\right)$$

$$\hat{B}_{ij} = \sum_\alpha B_\alpha^{ij} = \sum_\alpha \int \frac{ds'}{\pi} \frac{g_\alpha^{(R)i} \rho_\alpha(s', m_{1\alpha}, m_{2\alpha}) g_\alpha^{(L)j}}{s' - s - i0}$$

**In the present fits we calculate the elements of the $B^{ij}_\alpha$ using one subtraction taken at the channel threshold $M_\alpha = (m_{1\alpha} + m_{2\alpha})$:**

$$B^{ij}_\alpha(s) = B^{ij}_\alpha(M^2_\alpha) + (s - M^2_\alpha) \int\limits_{m^2_a}^{\infty} \frac{ds'}{\pi} \frac{g^{(R)i}_\alpha \rho_\alpha(s', m_{1\alpha}, m_{2\alpha}) g^{(L)j}_\alpha}{(s' - s - i0)(s' - M^2_\alpha)} .$$

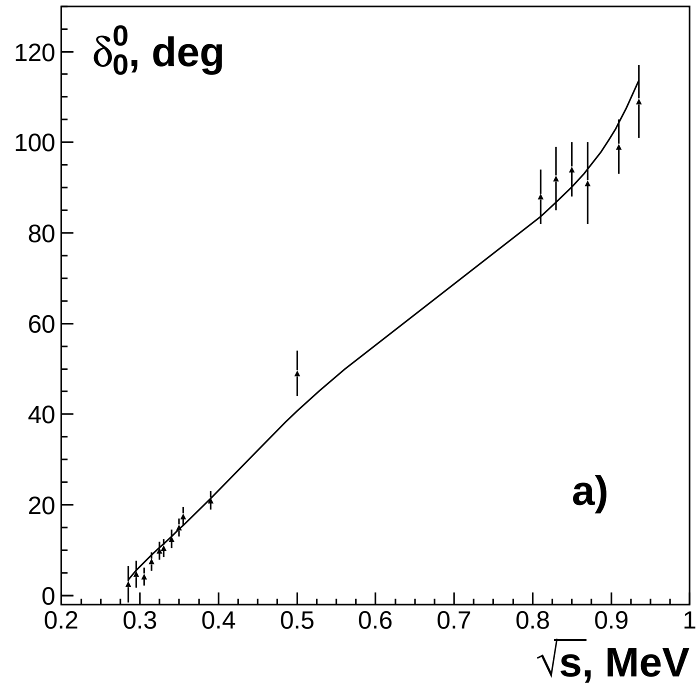
**In this case the expression for elements of the $\hat{B}$ matrix can be rewritten as:**

$$B^{ij}_\alpha(s) = g^{(R)i}_a \left( b^\alpha + (s - M^2_\alpha) \int\limits_{m^2_a}^{\infty} \frac{ds'}{\pi} \frac{\rho_\alpha(s', m_{1\alpha}, m_{2\alpha})}{(s' - s - i0)(s' - M^2_\alpha)} \right) g^{(L)j}_\beta = g^{(R)i}_a B_\alpha g^{(L)j}_\beta$$

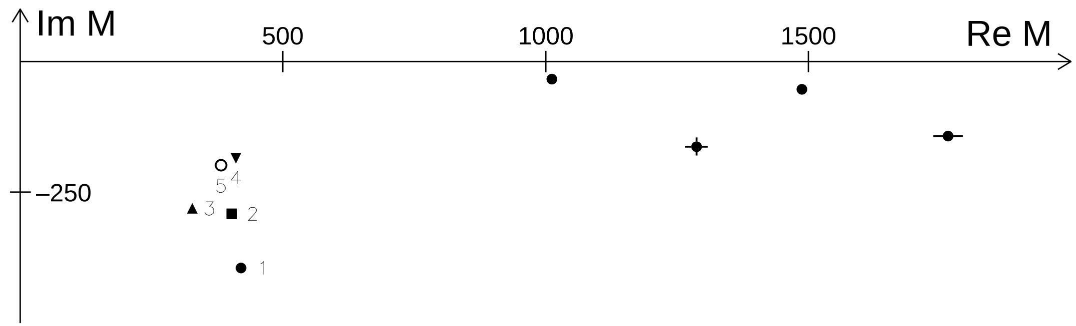
**and D-matrix method equivalent to the K-matrix method with loop diagram with real part taken into account:**

$$A = \hat{K}(I - \hat{B}\hat{K})^{-1} \qquad B_{\alpha\beta} = \delta_{\alpha\beta} B_\alpha$$

# Pole position of the resonances




| | K-matrix | D-matrix |
|---|---|---|
| $\sigma$-meson | 420-i 395 | 407-i 281 |
| $f_0(980)$ | 1014-i 31 | 1015-i 36 |
| $f_0(1300)$ | 1302-i 140 | 1307-i 137 |
| $f_0(1500)$ | 1487-i 58 | 1487-i 60 |
| $f_0(1750)$ | 1738-i 152 | 1781-i 140 |




$445^{+16}_{-8} - i272^{+9}_{-13}$ **I.Caprini, G.Colangelo, and H.Leutwyler, Phys.Rev.Lett.96, 132001 (2006)**

$457^{+14}_{-15} - i279^{+11}_{-7}$ **R.Garcia-Martin, R.Kaminski, J.R.Pelaez, J.Ruiz de Elvira, and F.J.Yndurain**

## Pole parameters of the $S_{11}$ states

| | $N(1535)S_{11}$ | | $N(1650)S_{11}$ | | $N(1890)S_{11}$ | |
|---|---|---|---|---|---|---|
| | K-matrix | D-matrix | K-matrix | D-matrix | K-matrix | D-matrix |
| $M_{\rm pole}$ | 1501±4 | 1494 | 1647±6 | 1651 | 1900±15 | 1905 |
| $\Gamma_{\rm pole}$ | 134±11 | 116 | 103±8 | 95 | $90^{+30}_{-15}$ | 106 |
| Elastic residue | 31±4 | 25 | 24±3 | 23 | 1±1 | 1.5 |
| Phase | -(29±5)$^o$ | -38$^o$ | -(75±12)$^o$ | -62$^o$ | – | – |
| Res$_{\pi N\to N\eta}$ | 28±3 | 25 | 15±3 | 15 | 4±2 | 5 |
| Phase | -(76±8)$^o$ | -69$^o$ | (132±10)$^o$ | 140 | (40±20)$^o$ | 42$^o$ |
| Res$_{\pi N\to \Delta\pi}$ | 7±4 | 4 | 11±3 | 12 | – | – |
| Phase | (147±17)$^o$ | 157$^o$ | -(30±20)$^o$ | -40 | – | – |
| $A^{1/2}$ (GeV$^{-\frac{1}{2}}$) | 0.116±0.010 | 0.107 | 0.033±0.007 | 0.029 | 0.012±0.006 | 0.010 |
| Phase | (7±6)$^o$ | 1$^o$ | -(9±15)$^o$ | 0$^o$ | 120±50$^o$ | 150$^o$ |

# Minimization methods

1. **The two body final states $\pi N, \gamma N \to \pi N, \eta N, K\Lambda, K\Sigma, \omega N, K^*\Lambda$: $\chi^2$ method.**

   **For $n$ measured bins we minimize**

$$\chi^2 = \sum_j^n \frac{\left(\sigma_j(PWA) - \sigma_j(exp)\right)^2}{(\Delta\sigma_j(exp))^2}$$

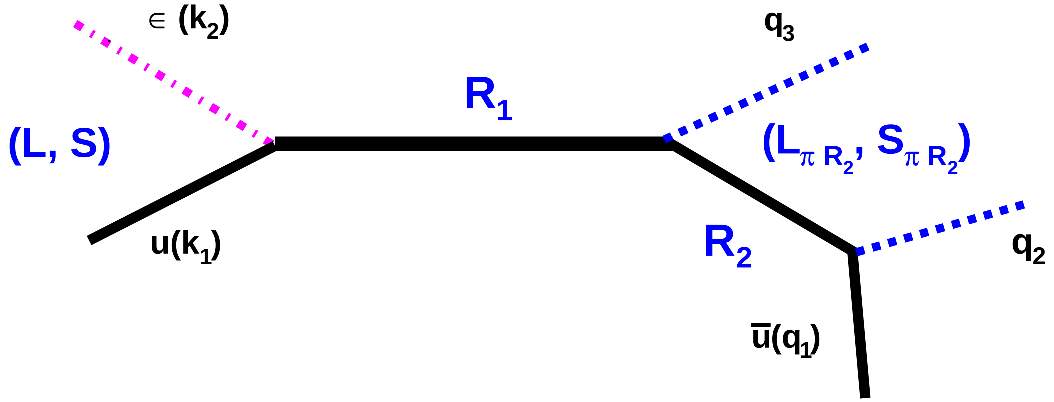
   **Present solution $\chi^2 = 48710$ for $31180$ points. $\chi^2/N_F = 1.6$**

2. **Reactions with three or more final states are analyzed with logarithm likelihood method. $\pi N, \gamma N \to \pi\pi N, \pi\eta N, \omega p, K^*\Lambda$. The minimization function:**

$$f = -\sum_j^{N(data)} ln \frac{\sigma_j(PWA)}{\sum\limits_m^{N(rec\,MC)} \sigma_m(PWA)}$$

   **This method allows us to take into account all correlations in many dimensional phase space. Above 500 000 data events are taken in the fit.**

# Resonance amplitudes for meson photoproduction




**General form of the angular dependent part of the amplitude:**

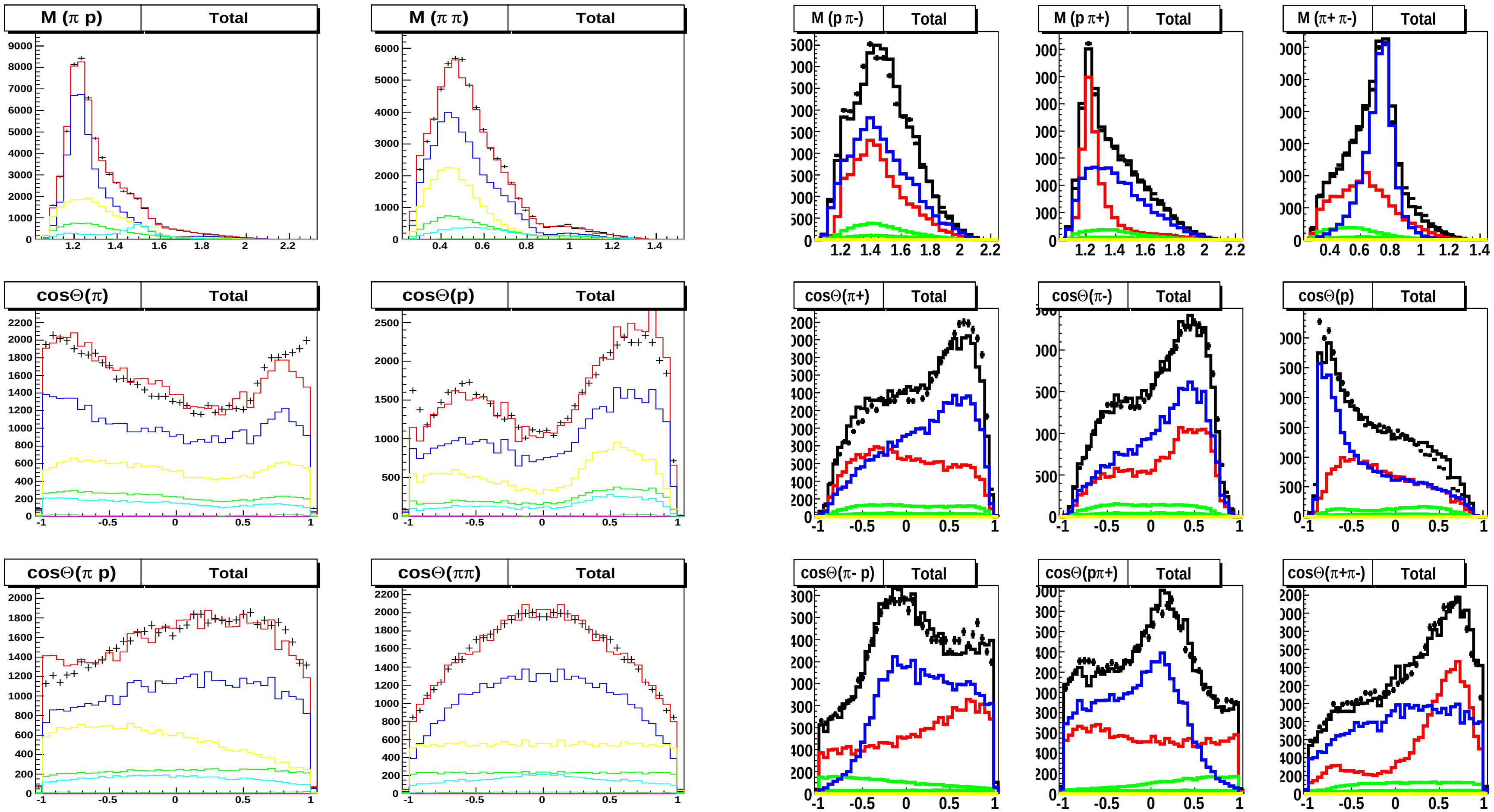
$$\bar{u}(q_1)\tilde{N}_{\alpha_1\ldots\alpha_n}(R_2\to\mu N)F^{\alpha_1\ldots\alpha_n}_{\beta_1\ldots\beta_n}(q_1+q_2)\tilde{N}^{(j)\beta_1\ldots\beta_n}_{\gamma_1\ldots\gamma_m}(R_1\to\mu R_2)$$
$$F^{\gamma_1\ldots\gamma_m}_{\xi_1\ldots\xi_m}(P)V^{(i)\mu}_{\xi_1\ldots\xi_m}(R_1\to\gamma N)u(k_1)\varepsilon_\mu$$

$$F^{\mu_1\ldots\mu_L}_{\nu_1\ldots\nu_L}(p)=(m+\hat{p})O^{\mu_1\ldots\mu_L}_{\alpha_1\ldots\alpha_L}\frac{L+1}{2L+1}\left(g^{\perp}_{\alpha_1\beta_1}-\frac{L}{L+1}\sigma_{\alpha_1\beta_1}\right)\prod_{i=2}^{L}g_{\alpha_i\beta_i}O^{\beta_1\ldots\beta_L}_{\nu_1\ldots\nu_L}$$

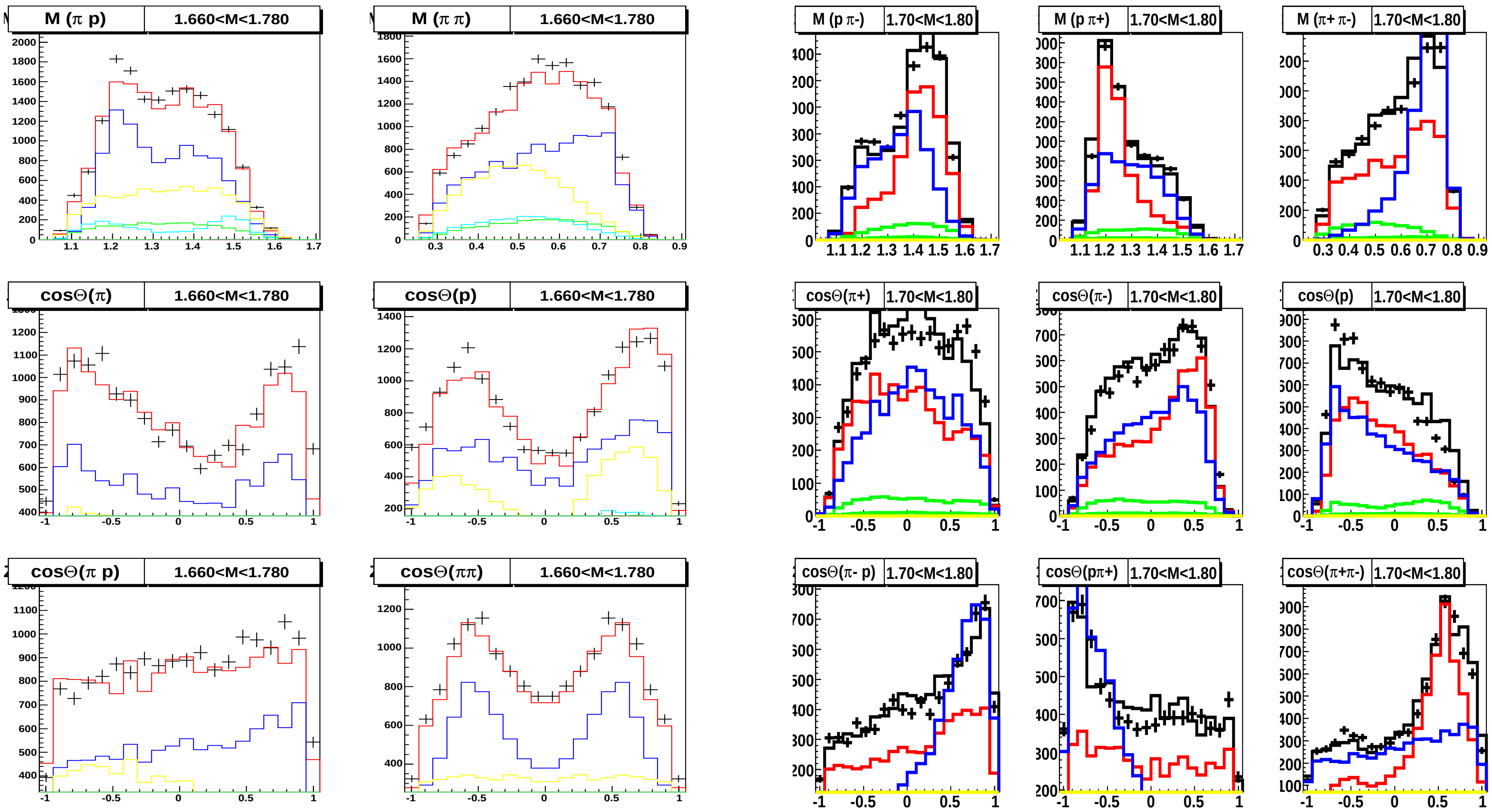
$$\sigma_{\alpha_i\alpha_j}=\frac{1}{2}(\gamma_{\alpha_i}\gamma_{\alpha_j}-\gamma_{\alpha_j}\gamma_{\alpha_i})$$

| DATA | BG2011-2014 | added in BG2015-2016 |
|---|---|---|
| $\pi N \to \pi N$ ampl. | SAID or Hoehler energy fixed | |
| $\gamma p \to \pi N$ | $\frac{d\sigma}{d\Omega}, \Sigma, T, P, E, G, H$ | $\frac{d\sigma}{d\Omega} E, G, T, P, H, F$ (CB-ELSA, CLAS, MAMI) |
| $\gamma n \to \pi N$ | $\frac{d\sigma}{d\Omega}, \Sigma, T, P$ | $\frac{d\sigma}{d\Omega}(MAMI)$, $\Sigma$ (CLAS) |
| $\gamma n \to \Lambda n, \Sigma^- p$ | - | $\frac{d\sigma}{d\Omega}$ |
| $\gamma n \to \eta n$ | $\frac{d\sigma}{d\Omega}, \Sigma$ | $\frac{d\sigma}{d\Omega}$ (MAMI) |
| $\gamma p \to \eta p$ | $\frac{d\sigma}{d\Omega}, \Sigma$ | $T, P, H, E, F$ |
| $\gamma p \to \eta' p$ | | $\frac{d\sigma}{d\Omega}, \Sigma$ |
| $\gamma p \to K^+ \Lambda$ | $\frac{d\sigma}{d\Omega}, \Sigma, P, T, C_x, C_z, O_{x'}, O_{z'}$ | $\Sigma, P, T, O_x, O_z$ (CLAS) |
| $\gamma p \to K^+ \Sigma^0$ | $\frac{d\sigma}{d\Omega}, \Sigma, P, C_x, C_z$ | $\Sigma, P, T, O_x, O_z$ (CLAS) |
| $\gamma p \to K^0 \Sigma^+$ | $\frac{d\sigma}{d\Omega}, \Sigma, P$ | |
| $\pi^- p \to \eta n$ | $\frac{d\sigma}{d\Omega}$ | |
| $\pi^- p \to K^0 \Lambda$ | $\frac{d\sigma}{d\Omega}, P, \beta$ | |
| $\pi^- p \to K^0 \Sigma^0$ | $\frac{d\sigma}{d\Omega}, P\ (K^0\Sigma^0)\ \frac{d\sigma}{d\Omega}\ (K^+\Sigma^-)$ | |
| $\pi^+ p \to K^+ \Sigma^+$ | $\frac{d\sigma}{d\Omega}, P, \beta$ | |
| $\pi^- p \to \pi^0 \pi^0 n$ | $\frac{d\sigma}{d\Omega}$ (Crystal Ball) | |
| $\pi^- p \to \pi^+ \pi^- n$ | | $\frac{d\sigma}{d\Omega}$ (HADES) |
| $\pi^- p \to \pi^- \pi^0 p$ | | $\frac{d\sigma}{d\Omega}$ (HADES) |
| $\gamma p \to \pi^0 \pi^0 p$ | $\frac{d\sigma}{d\Omega}, \Sigma, E, I_c, I_s$ | $T, P, H, F, P_x, P_y$ |
| $\gamma p \to \pi^0 \eta p$ | $\frac{d\sigma}{d\Omega}, \Sigma, I_c, I_s$ | |
| $\gamma p \to \pi^+ \pi^- p$ | | $\frac{d\sigma}{d\Omega}, I_c, I_s$ (CLAS) |
| $\gamma p \to \omega p$ | | $\frac{d\sigma}{d\Omega}, \Sigma, \rho^0_{ij}, \rho^1_{ij}, \rho^2_{ij}, E, G$ (CB-ELSA) |

# The description of the $\gamma p \to \pi^0\pi^0 p$ and $\gamma p \to \pi^+\pi^- p$ data (preliminary)



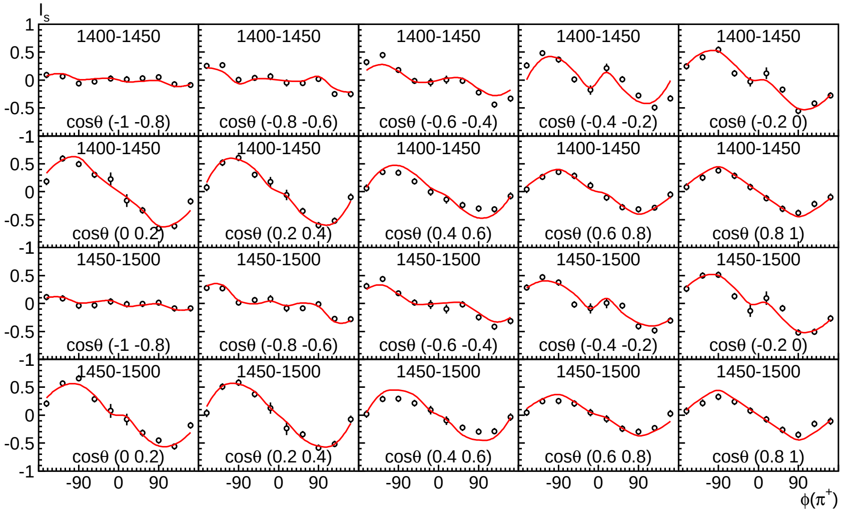
# The description of the $\gamma p \to \pi^0\pi^0 p$ and $\gamma p \to \pi^+\pi^- p$ data for 1700 MeV$< W$,1800 MeV (preliminary)



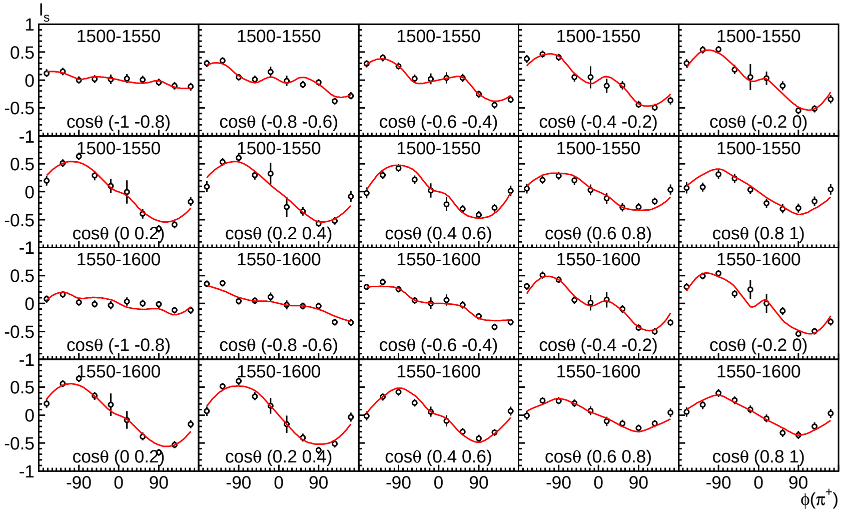
# $I_c$ and $I_s$ for $\gamma p \to \pi^+\pi^- p$ from CLAS (Preliminary)

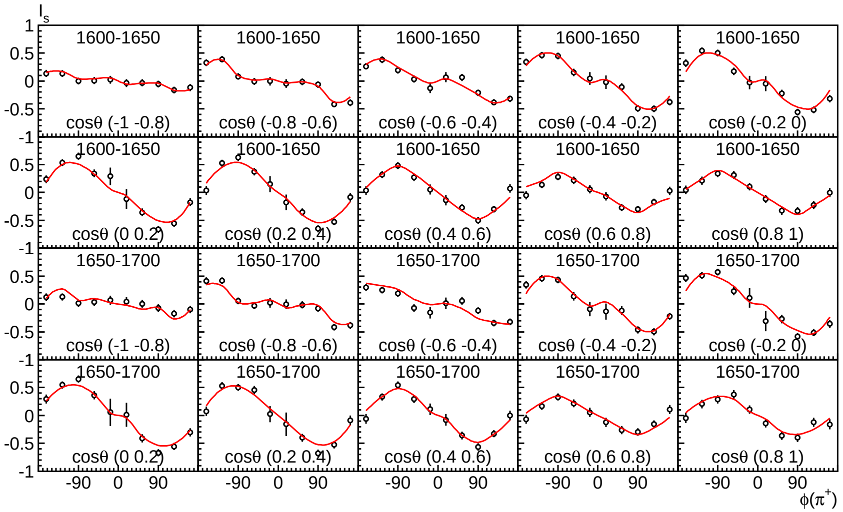
## Courtesy of V. Crede, Florida State U

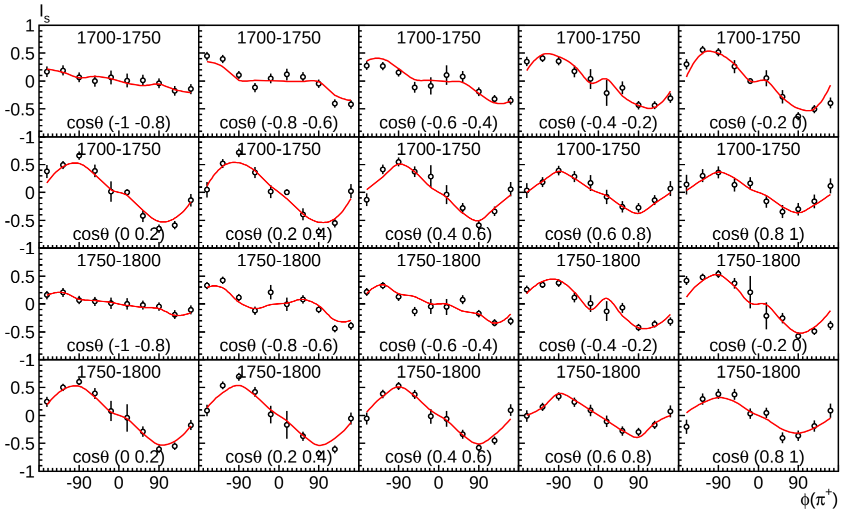
$I_s$



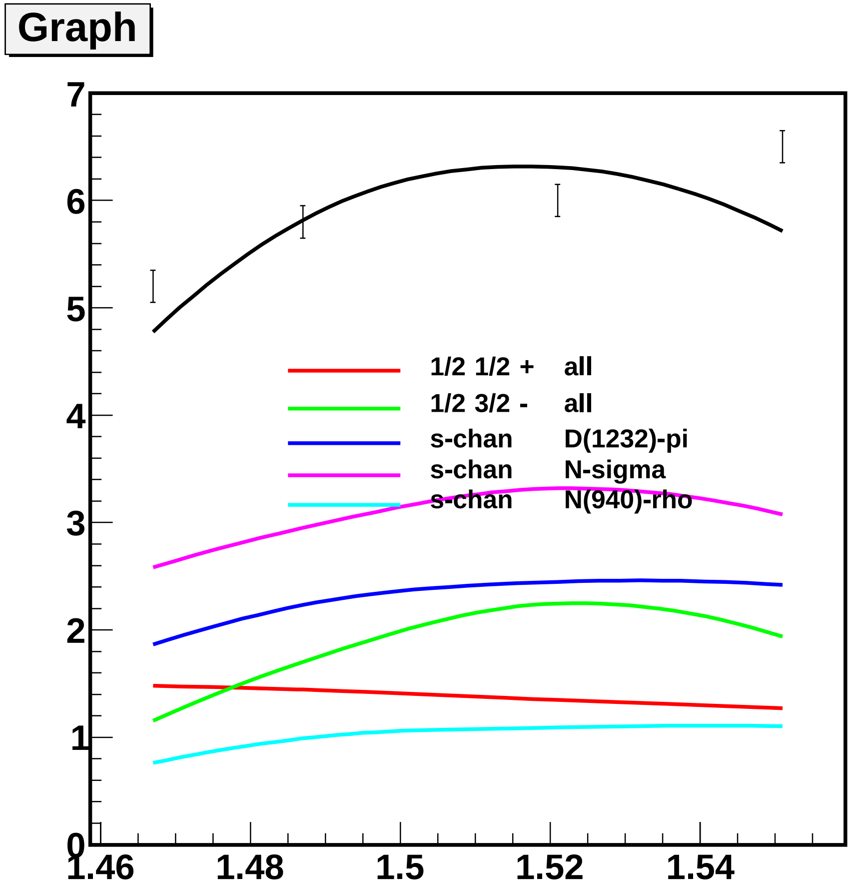

$I_s$

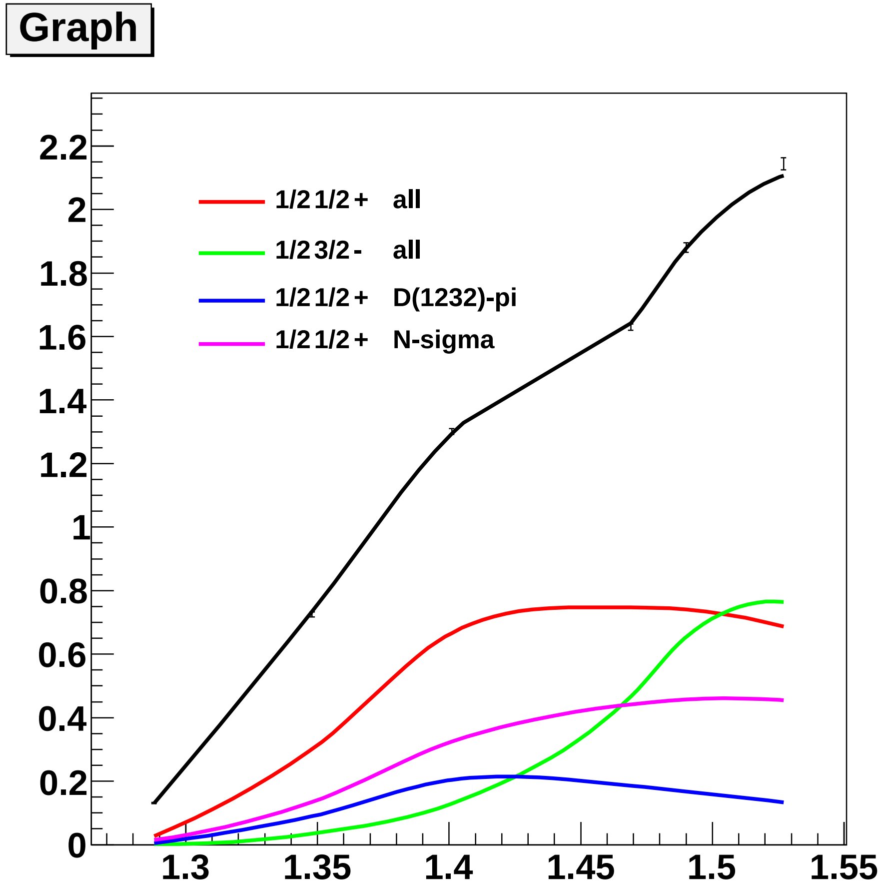


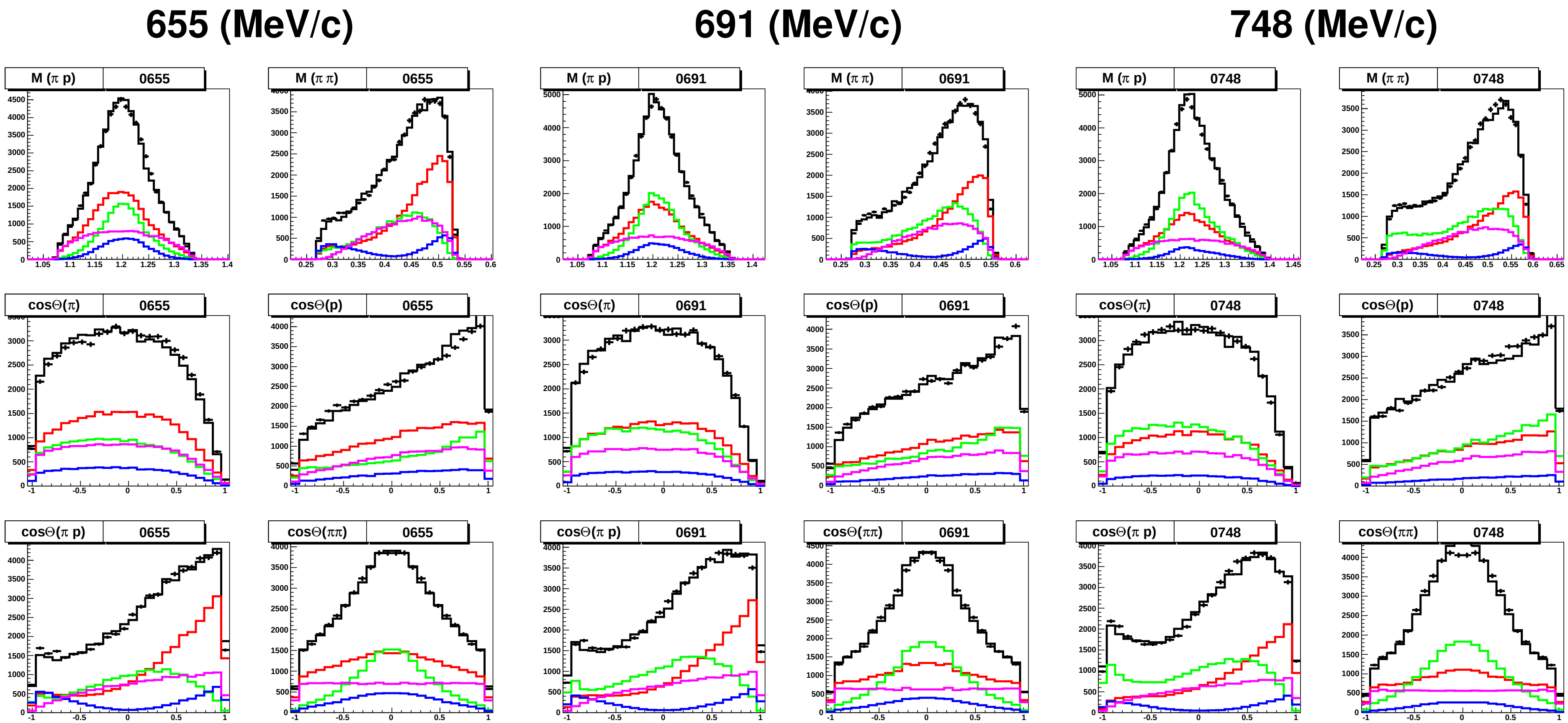



# The total cross section from the $\pi^- p \to \pi^+\pi^- n$ and $\pi^- p \to 2\pi^0 n$ data



# Crystal Ball data on $\pi^- p \to \pi^0 \pi^0 n$




1/2 1/2+ 1/2 3/2-

# Meson production in NN collision

$$A=\left(\bar u(p_1')V^{S',L'}_{\mu_1\ldots\mu_J}(k'_\perp)u^c(-p_2')\right)O^{\mu_1\ldots\mu_n}_{\nu_1\ldots\nu_n}\left(\bar u^c(-p_2)G^{J,P}_{\nu_1\ldots\nu_J}u(p_1)\right)A_{pw}(s)\,.$$

**Let us consider the transition from the $NN$ state with $J^P$ into a pseudoscalar meson with momentum $k_1$ and $NN$ system with momenta $k_2, k_3$ in state with $S', L', j, P'$ .**

**For $P=P'(-1)^{J+j+1}$ we have the following vertex:**

$$G^{J,P}_{\mu_1\ldots\mu_J}=V^{S',L'}_{\mu_1\ldots\mu_k\nu_{k+1}\ldots\nu_j}(k_{23})O^{\nu_{k+1}\ldots\nu_j\mu_{k+1}\mu_J}_{\alpha_1\ldots\alpha_{J+j-2k}}(k_2+k_3)X^{(L)}_{\alpha_1\ldots\alpha_{J+j-2k}}(k_1^\perp),$$

**Here $k=0,\ldots j$ and $L=J+j-2k\geq 0$.**

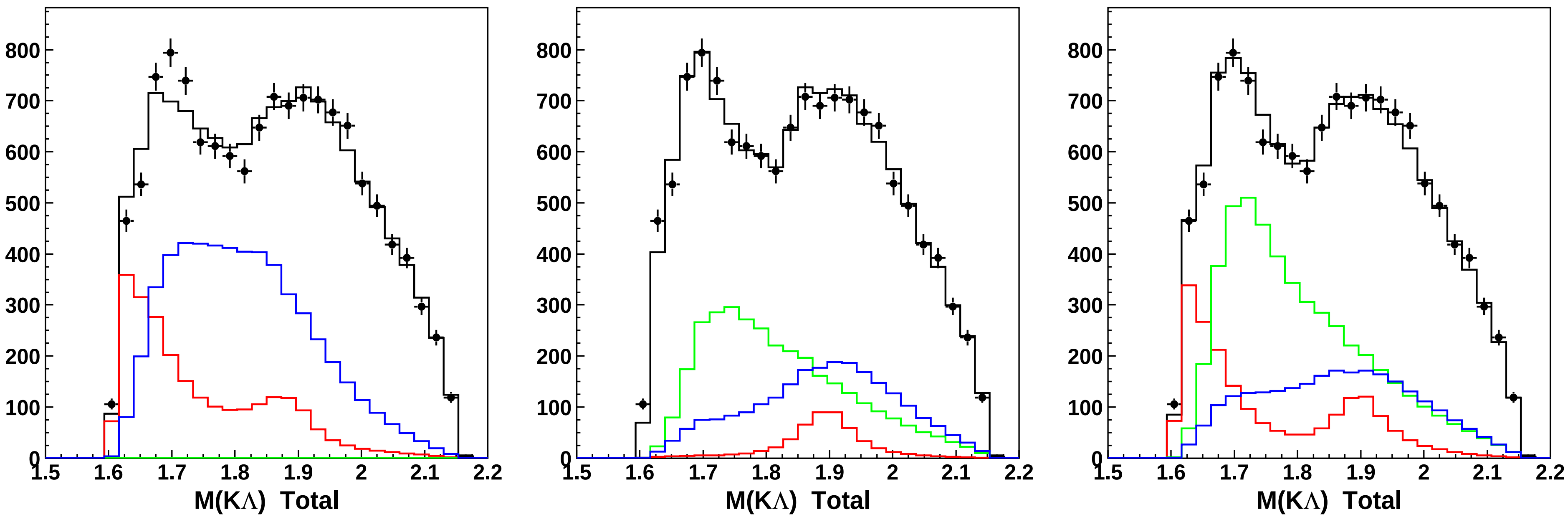
**For $P=P'(-1)^{J+j}$:**

$$\begin{aligned}G^{J,P}_{\mu_1\ldots\mu_J} \quad = \quad & \varepsilon_{\mu_1\alpha\beta\eta}V^{S',L'}_{\alpha\mu_2\ldots\mu_k\nu_{k+1}\ldots\nu_j}(k_{23})O^{\nu_{k+1}\ldots\nu_j\mu_{k+1}\ldots\mu_J}_{\alpha_1\ldots\alpha_{J+j-2k+1}}(k_2+k_3)\times\\ & X^{(L)}_{\beta\alpha_1\ldots\alpha_{J+j-2k+1}}(k_1^\perp)P_\eta,\end{aligned}$$

**Here $k=1,\ldots j$ and $L=J+j-2k+1\geq 0$.**

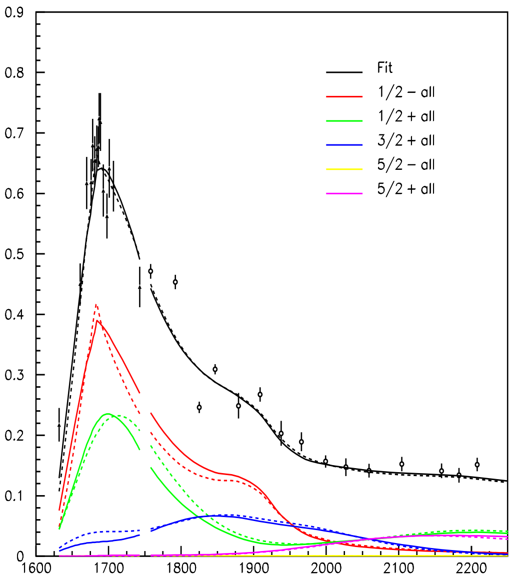
# Partial wave analysis of HADES $pp \to K^+\Lambda p$ data

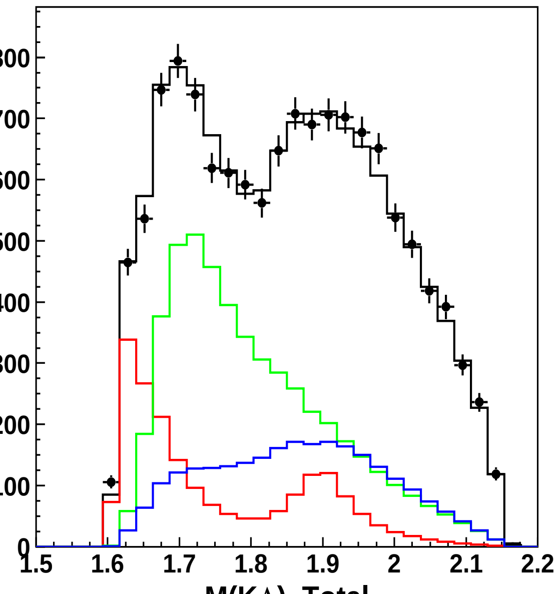
**No** $P_{11}(1710)$ **No** $S_{11}(1650)$ **Both are included**




$S_{11}(1/2^-)$ $P_{11}(1/2+)$ $P_{13}(3/2+)$

# Partial wave contributions to $\pi^- p \to K\Lambda$ and $pp \to K^+\Lambda p$




| $\pi N + \gamma N$ | $pp \to K^+\Lambda p$ |
|---|---|
| $P_{11}(1710)$ | |
| $1690 \pm 10$ | $1692 \pm 9$ |
| $168 \pm 27$ | $170 \pm 20$ |
| $S_{11}(1895)$ | |
| $1891 \pm 7$ | $1907 \pm 15$ |
| $84 \pm 22$ | $100^{+40}_{-15}$ |
| $P_{13}(1900)$ | |
| $1906 \pm 19$ | $1910 \pm 30$ |
| $290 \pm 55$ | $280 \pm 50$ |

$S_{11}(1/2^-)$ $P_{11}(1/2+)$ $P_{13}(3/2+)$

**For HADES $pp \to K^+\Lambda p$ only systematic errors are given.**

**The following data sets were analyzed in the framework of event-by-event maximum likelihood approach:**

| $n$ | Reaction | $p_{beam}$ | $N_{data}$ | Origin |
|---|---|---|---|---|
| 1 | $pp \to \pi^0 pp$ | 1683 MeV/c | 1094 | Gatchina |
| 2 | $pp \to \pi^0 pp$ | 1581 MeV/c | 903 | Gatchina |
| 3 | $pp \to \pi^0 pp$ | 1536 MeV/c | 1319 | Gatchina |
| 4 | $pp \to \pi^0 pp$ | 1485 MeV/c | 997 | Gatchina |
| 5 | $pp \to \pi^0 pp$ | 1437 MeV/c | 918 | Gatchina |
| 6 | $pp \to \pi^0 pp$ | 1389 MeV/c | 996 | Gatchina |
| 7 | $pp \to \pi^0 pp$ | 1341 MeV/c | 883 | Gatchina |
| 8 | $pp \to \pi^0 pp$ | 1279 MeV/c | 621 | Gatchina |
| 9 | $pp \to \pi^0 pp$ | 1217 MeV/c | 544 | Gatchina |
| 10 | $np \to \pi^- pp$ | 1-1.9 GeV/c | 8210 | Gatchina |
| 11 | $pp \to \pi^0 pp$ | 950 MeV/c | 154972 | Tübingen |
| 12 | $pp \to \pi^+ pn$ | 2032 MeV/c | 7902 | Tübingen |
| 13 | $pp \to \pi^0 pp$ | $\sigma_{tot}$ 1217-1683 MeV | 9 | Gatchina |

# Parameterization

$$d\sigma = \frac{(2\pi)^4|A|^2}{4|\vec{k}|\sqrt{s}}\, d\Phi_3(P, q_1, q_2, q_3)\ ,$$

$$A = \sum_\alpha A^\alpha_{tr}(s) Q^{in}_{\mu_1\ldots\mu_J}(SLJ) A_{2b}(i, S_2L_2J_2)(s_i) Q^{fin}_{\mu_1\ldots\mu_J}(i, S_2L_2J_2S'L'J)\ .$$

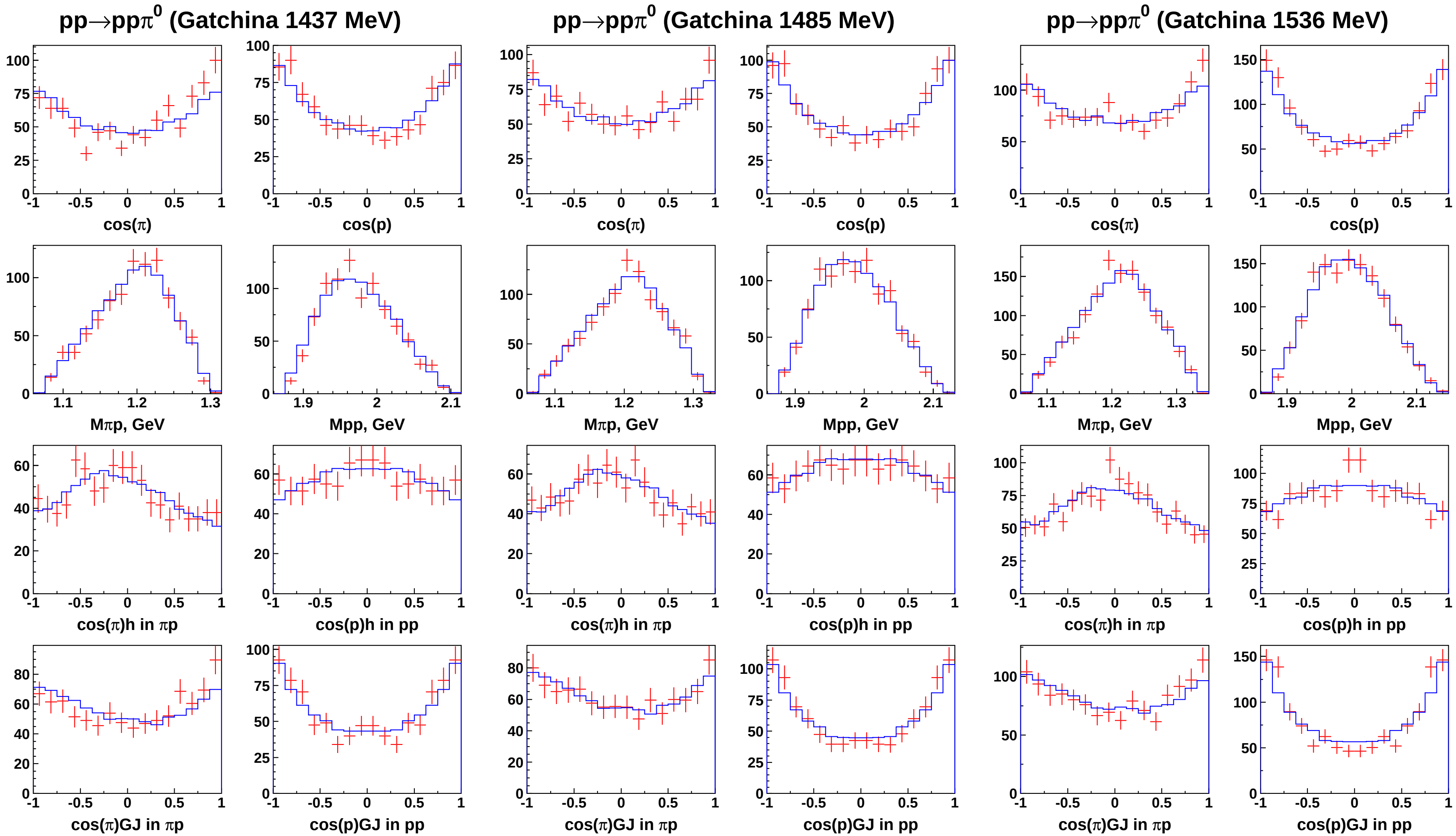
**Angular-spin momentum operators $Q_{\mu_1\ldots\mu_J}(SLJ)$ are given in A. V. Anisovich et. al Eur.Phys.J. A34 (2007) 129.**

$$A^\alpha_{tr}(s) = \frac{a^\alpha_1 + a^\alpha_3\sqrt{s}}{s - a^\alpha_4}\, e^{ia^\alpha_2},$$

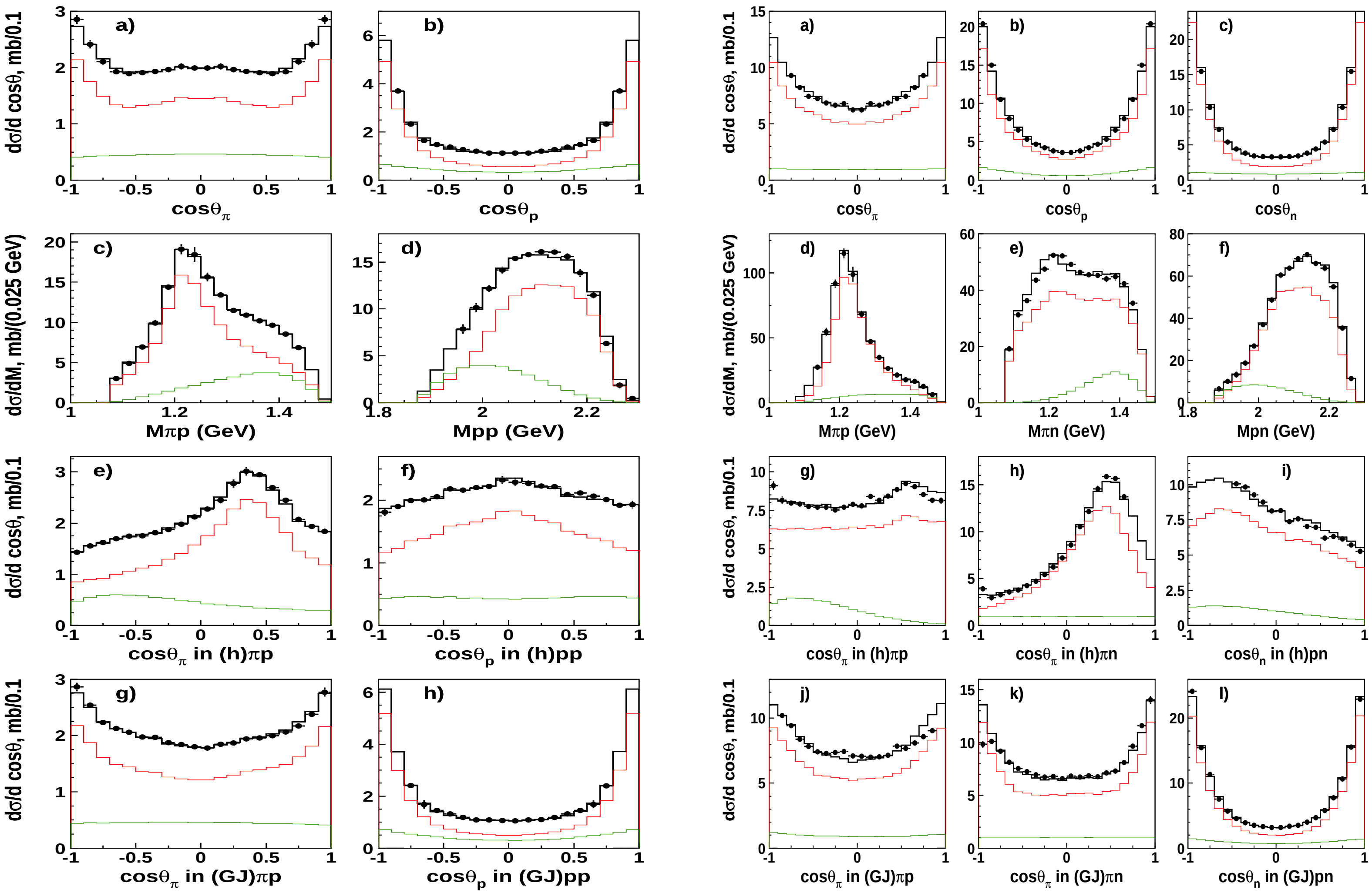
**Decay modes: $\Delta(1232)N$, $P_{11}(1440)N$ and $\pi(NN)$. In $NN$ channel amplitude was parameterized with generalized Watson-Migdal formula:**

$$A^\beta_{2b}(s_i) = \frac{\sqrt{s_i}}{1 - \frac{1}{2}r^\beta q^2 a^\beta_{pp} + iqa^\beta_{pp}q^{2L}/F(q, r^\beta, L)},$$

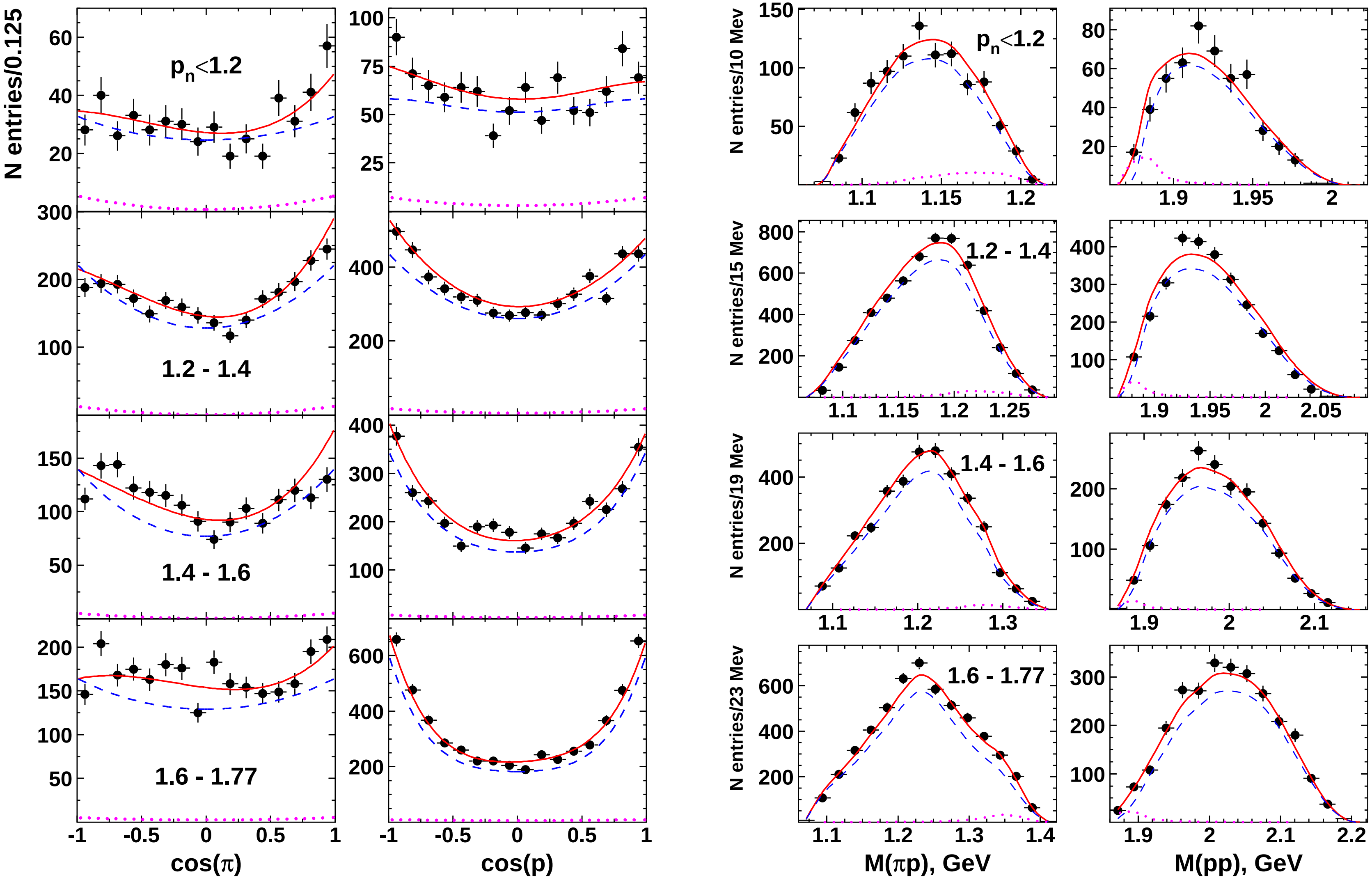
# Description of $pp \to pp\pi^0$:



# Description of the HADES data $pp \to pp\pi^0$ and $pp \to pn\pi^+$:



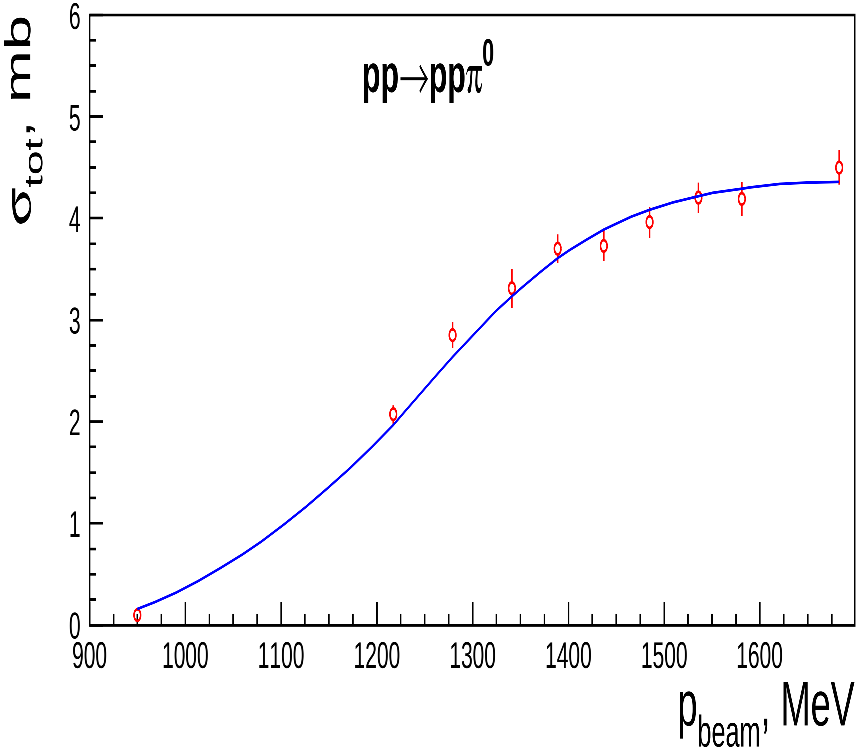
# $np \to \pi^- pp$



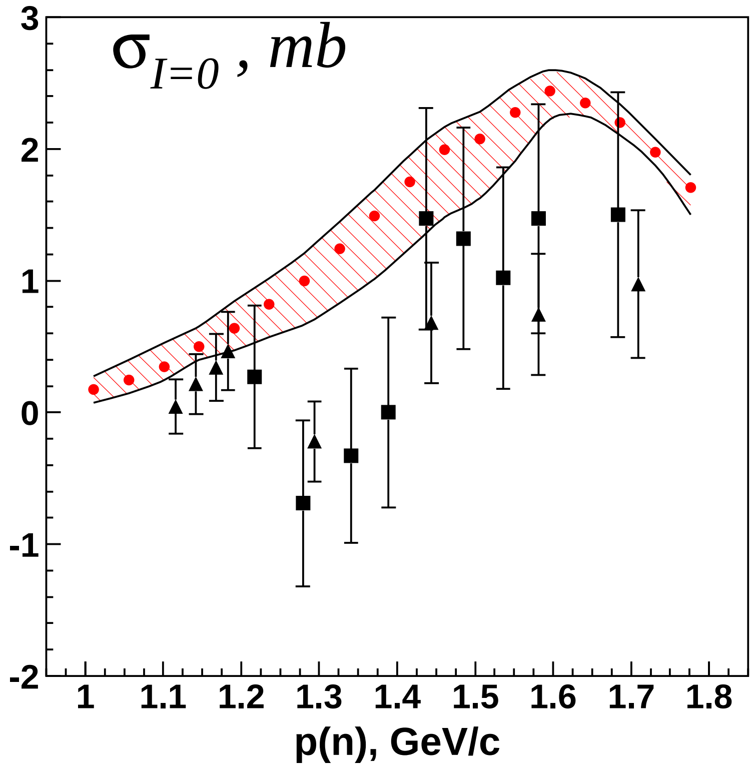

**Dashed** lines - $I = 1$, dotted lines - $I = 0$

**The cross section for pion production in nucleon-nucleon collision with $I = 1$ is well known. However there are very poor data about $I = 0$ cross section.**

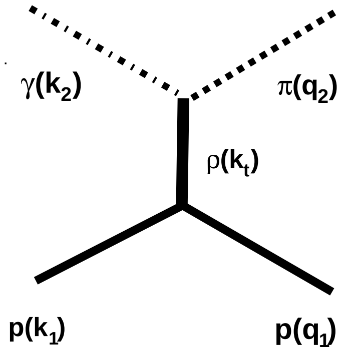
$$\sigma(I = 1) = \sigma(pp \to pp\pi^0)$$

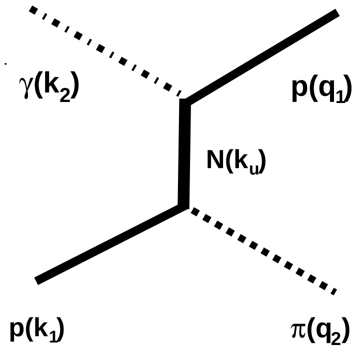



$$\sigma(I = 0) = 3[2\sigma(np \to pp\pi^-) \\ -\sigma(pp \to pp\pi^0)]$$



# Reggezied exchanges:




**The amplitude for t-channel exchange:**

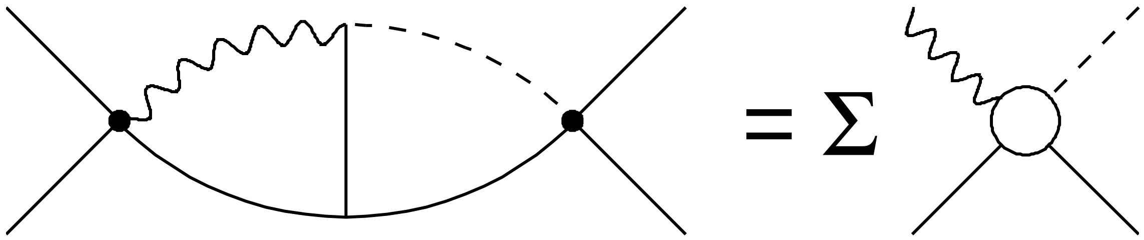
$$A = g_1(t)g_2(t)R(\xi,\nu,t) = g_1(t)g_2(t)\frac{1+\xi exp(-i\pi\alpha(t))}{\sin(\pi\alpha(t))}\left(\frac{\nu}{\nu_0}\right)^{\alpha(t)} \qquad \nu = \frac{1}{2}(s-u).$$

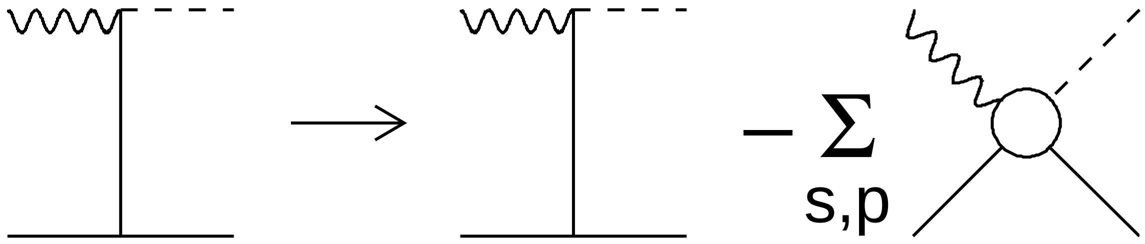
**Here $\alpha(t)$ is the reggion trajectory, and $\xi$ is its signature:**

$$R(+,\nu,t) = \frac{e^{-i\frac{\pi}{2}\alpha(t)}}{\sin(\frac{\pi}{2}\alpha(t))\Gamma\left(\frac{\alpha(t)}{2}\right)}\left(\frac{\nu}{\nu_0}\right)^{\alpha(t)},$$

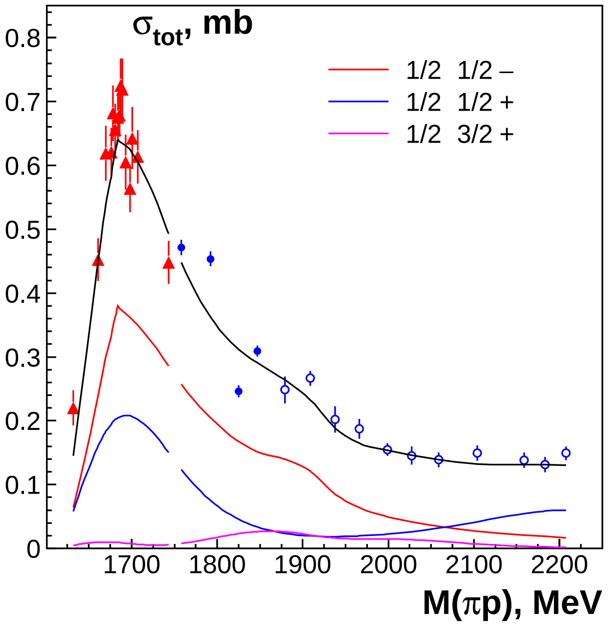
$$R(-,\nu,t) = \frac{ie^{-i\frac{\pi}{2}\alpha(t)}}{\cos(\frac{\pi}{2}\alpha(t))\Gamma\left(\frac{\alpha(t)}{2}+\frac{1}{2}\right)}\left(\frac{\nu}{\nu_0}\right)^{\alpha(t)}.$$

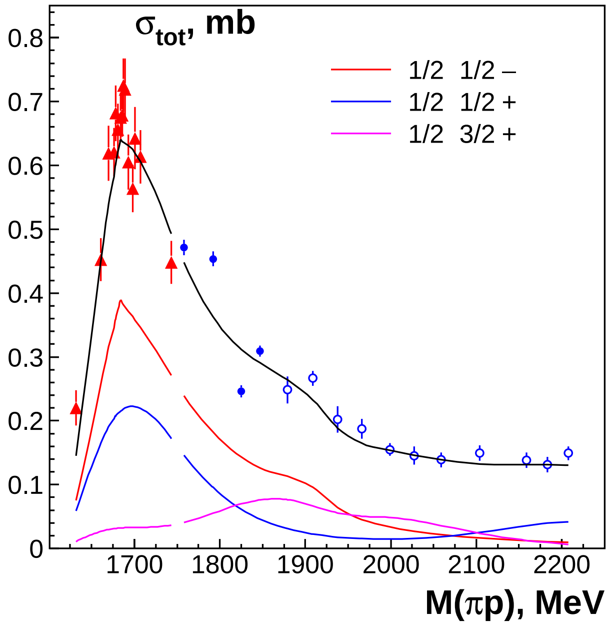
# t,u-exchange subtraction procedure

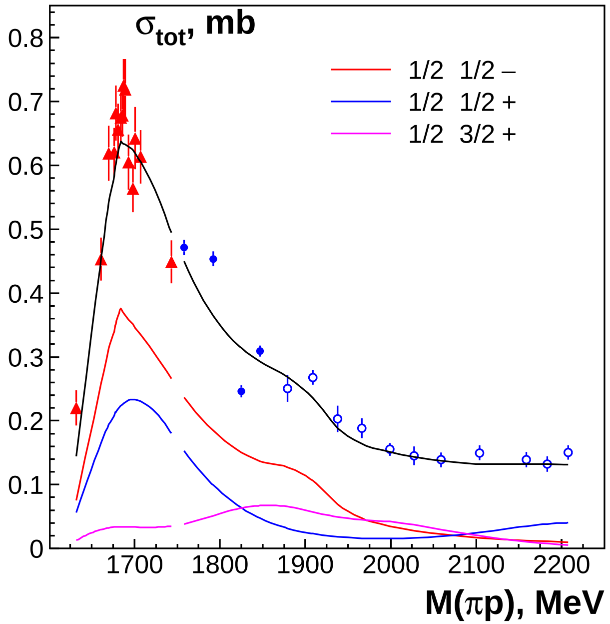


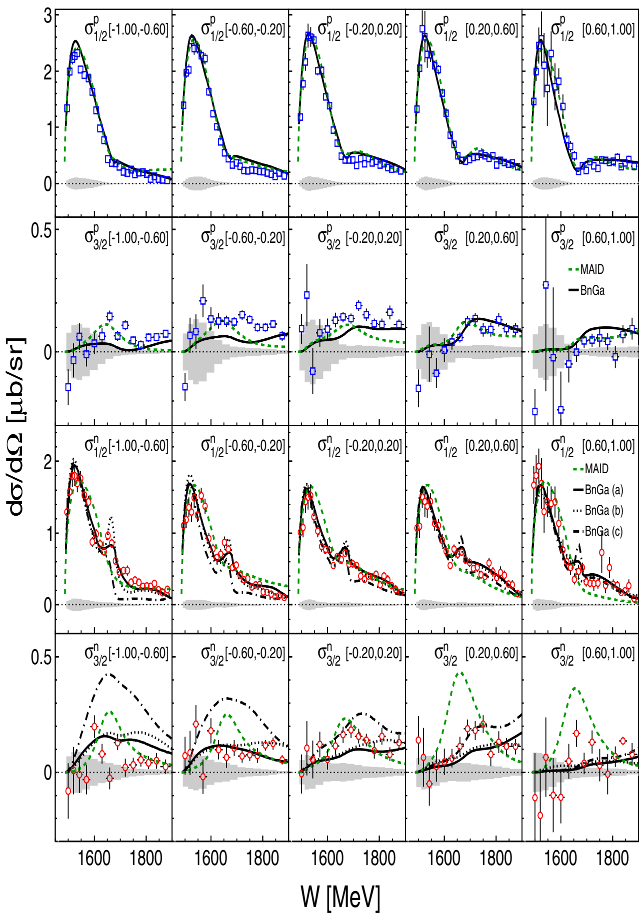
# t,u-exchange subtraction procedure

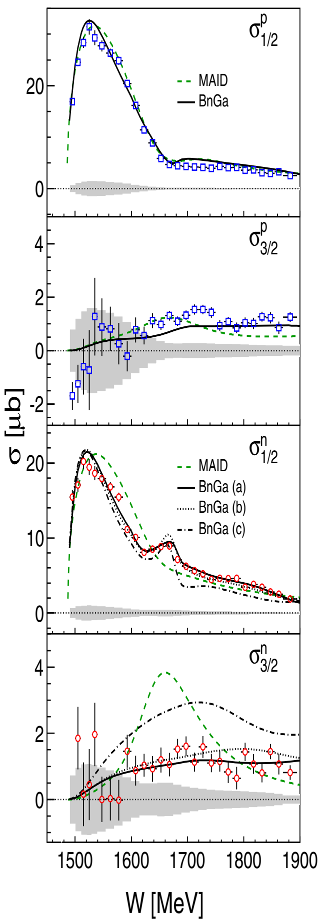


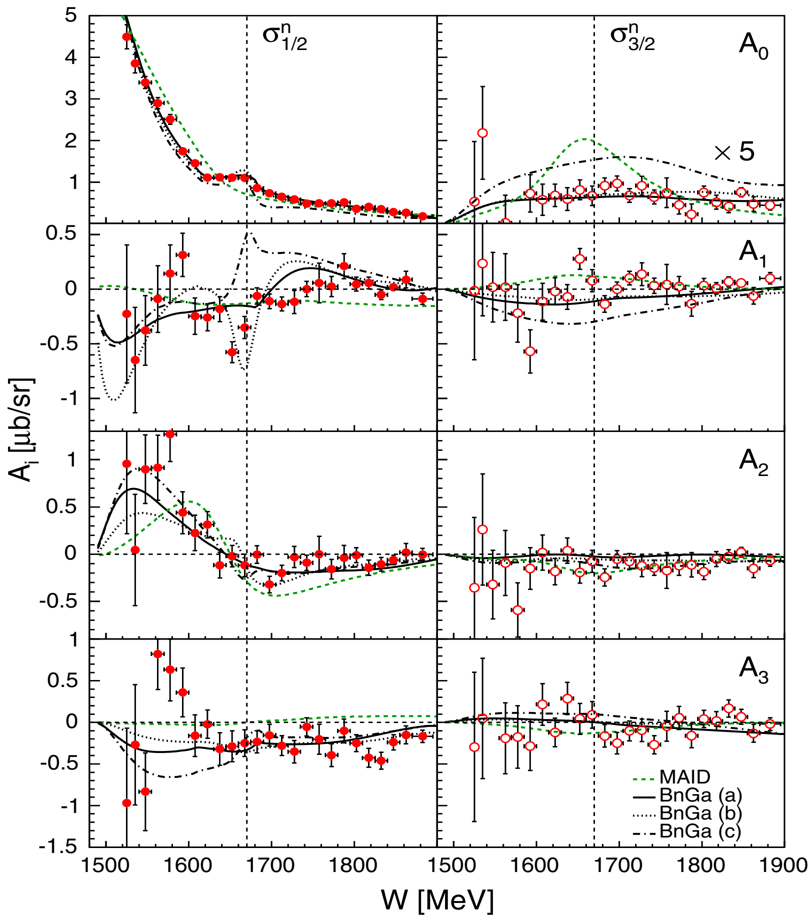



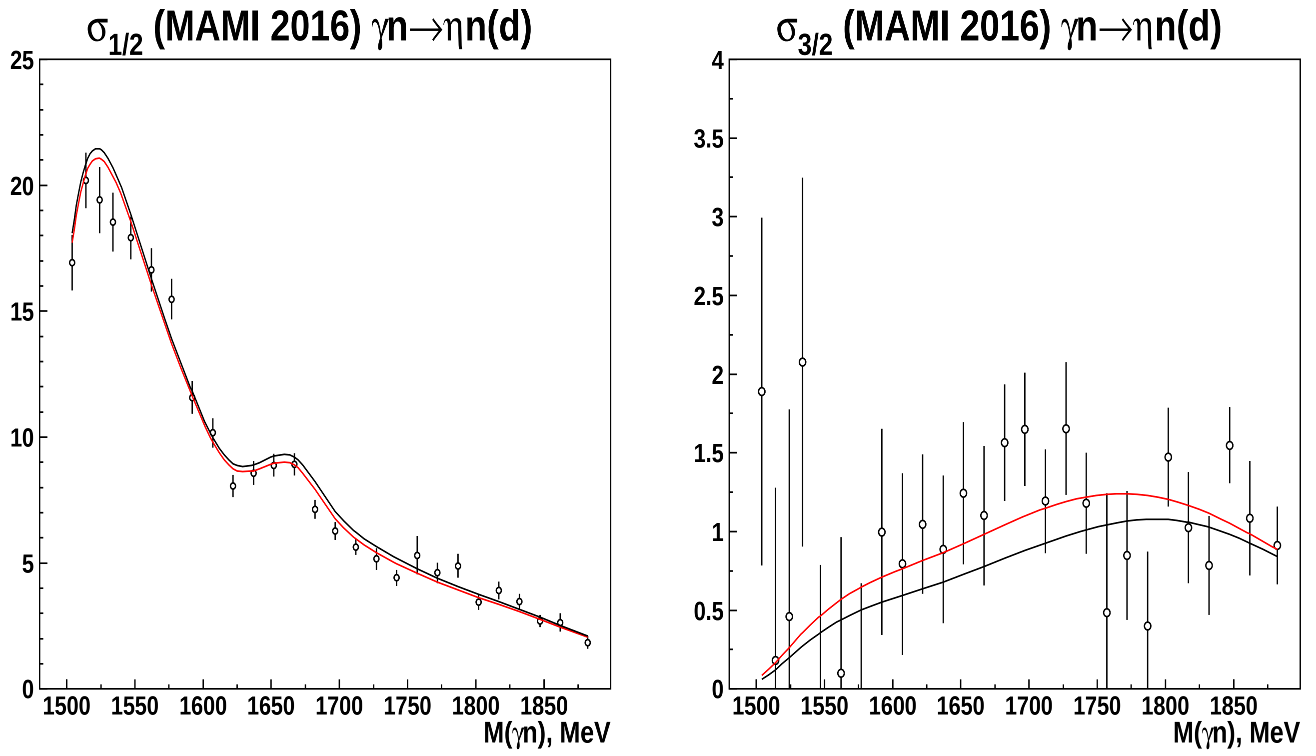
# The new MAMI data on helicity 1/2 and 3/2 cross section



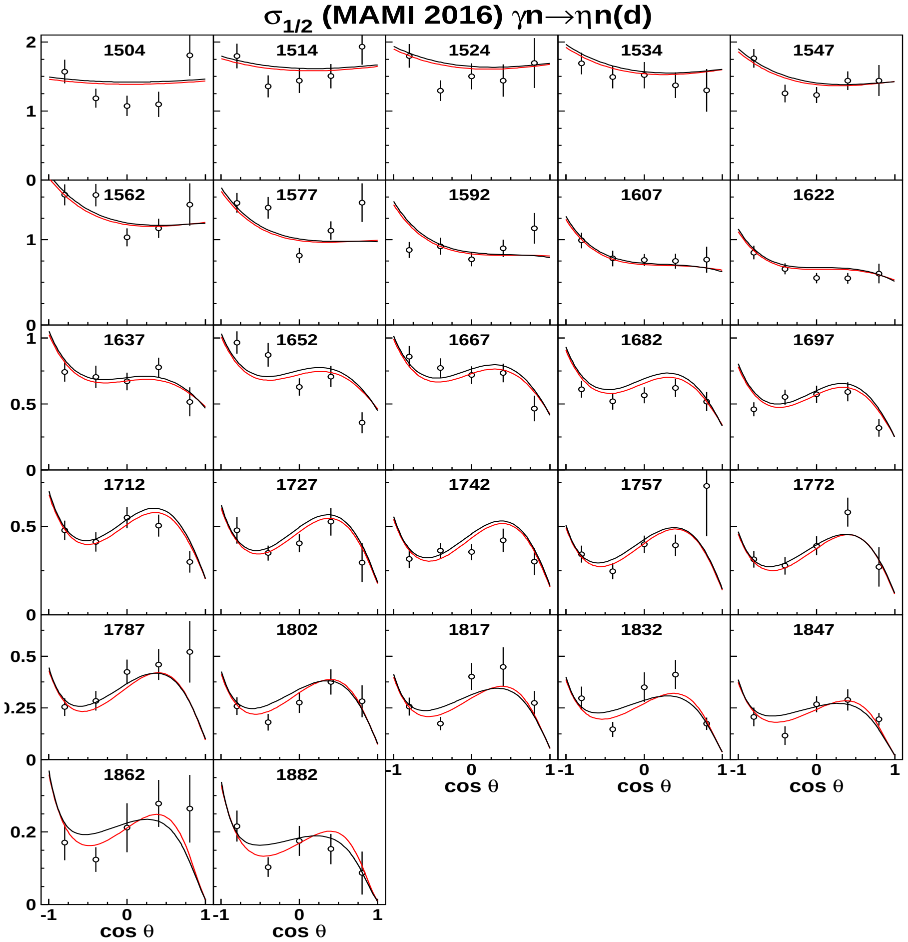



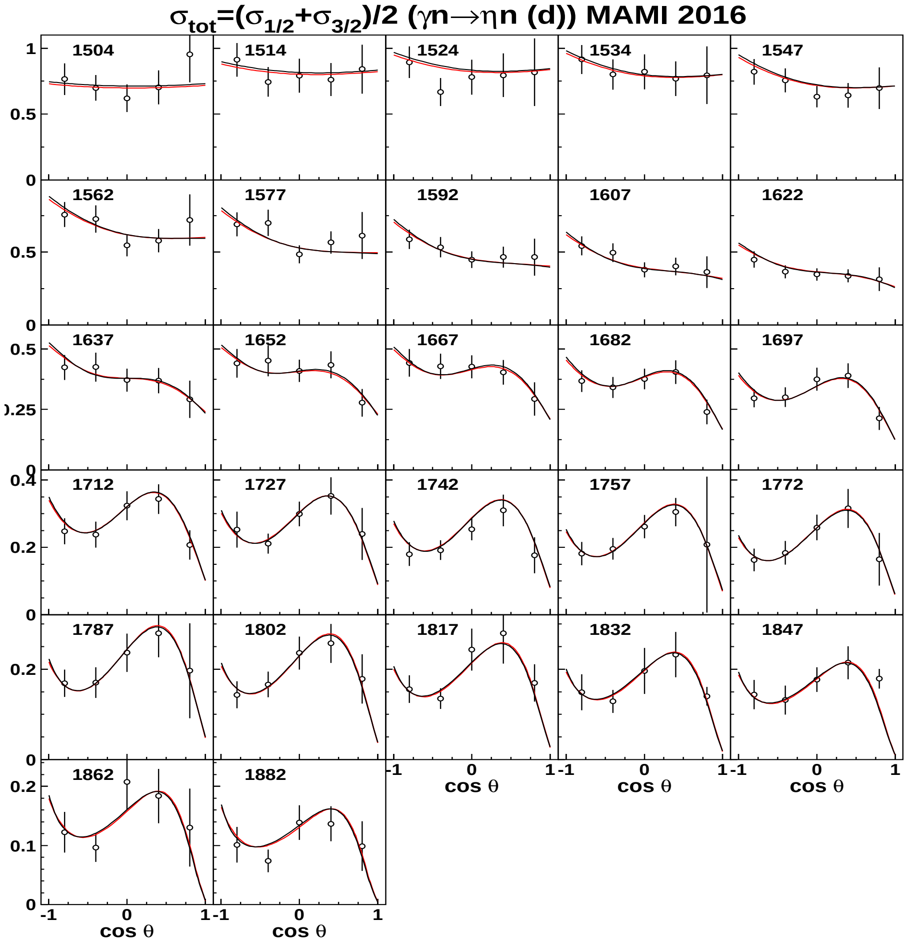
# The solution with the fitted $\gamma n \to K\Lambda, K\Sigma$ data




**Prediction** **Fit**

# The solution with the fitted $\gamma n \to K\Lambda, K\Sigma$ data




**Prediction**

**Fit**

# Vector mesons in the final state: Density matrices

$$\frac{d\sigma}{d\Omega_\omega\, d\Omega_{dec}} = \frac{d\sigma}{d\Omega_\omega} W(\cos\Theta_{dec}, \Phi_{dec})$$

$$\gamma p \to p\omega(\pi^+\pi^-\pi^0)$$

$$W(\cos\Theta,\Phi) = \frac{3}{4\pi}\Bigg(\frac{1}{2}(1-\rho_{00}) + \frac{1}{2}(3\rho_{00}-1)\cos^2\Theta - \sqrt{2}Re\rho_{10}\sin 2\Theta\cos\Phi - \rho_{1-1}\sin^2\Theta\cos 2\Phi\Bigg)\,.$$

$\cos\Theta, \Phi$ **direction of the vector** $n = \varepsilon_{ijkm} p_j^{\pi^+} p_k^{\pi^-} p_m^{\pi^0}$ **in the** $\omega$ **rest frame.**

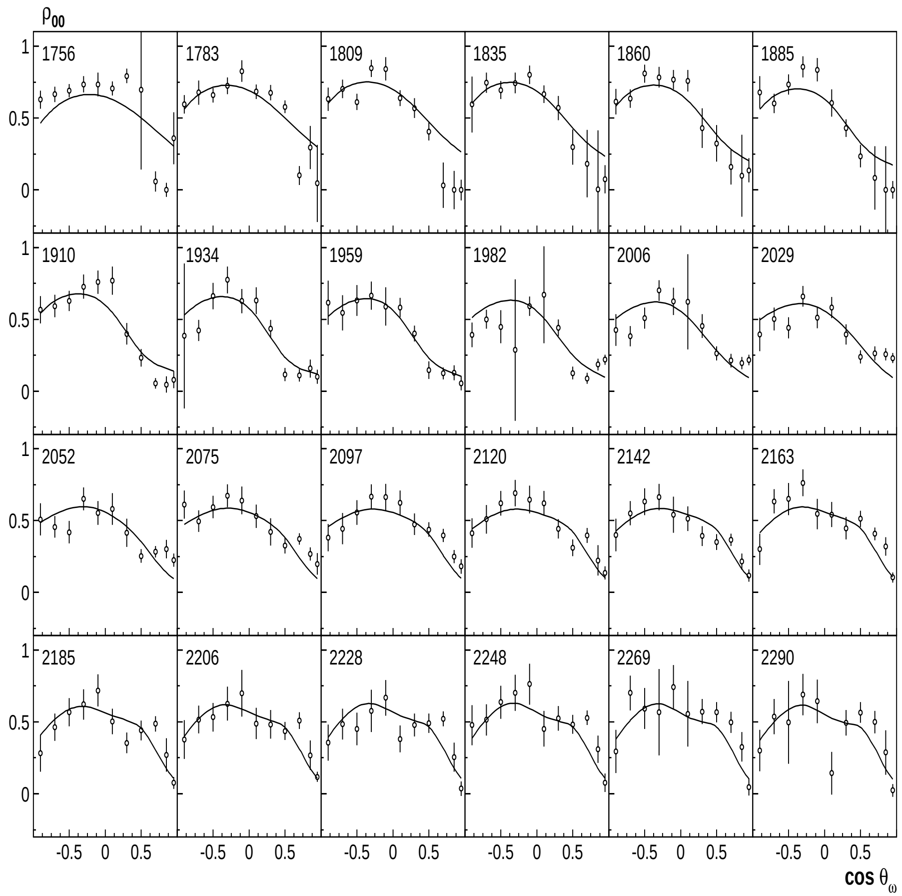
$$\gamma p \to p\omega(\gamma\pi^0)$$

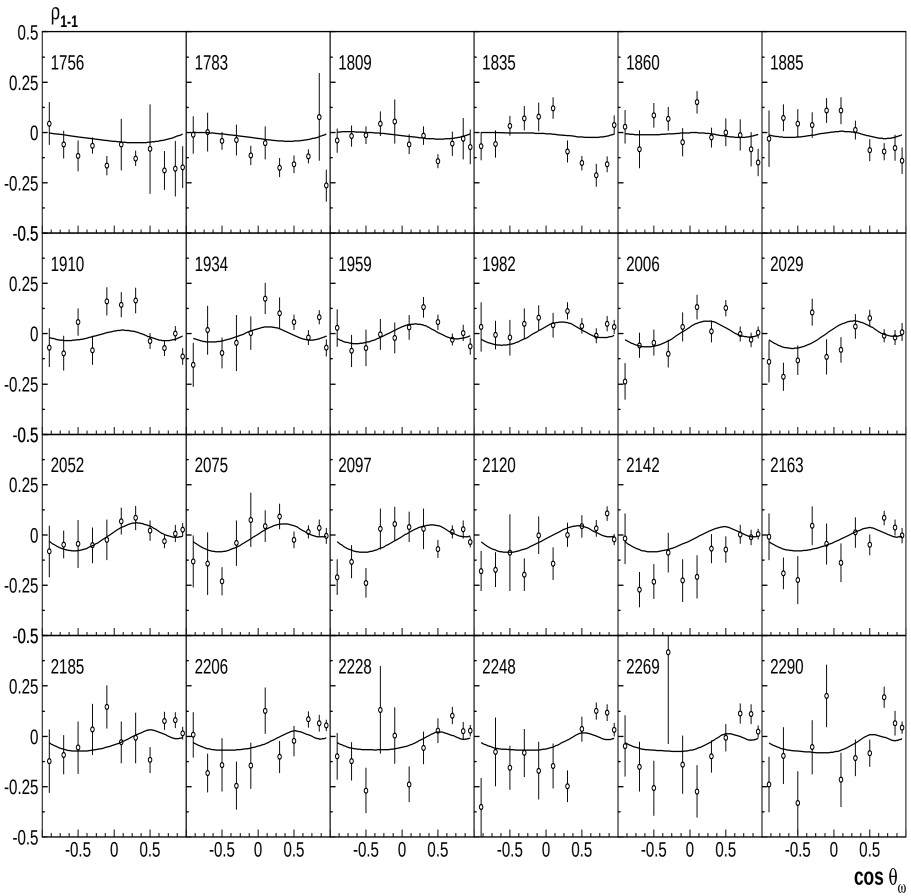
$$W(\cos\Theta,\Phi) = \frac{3}{8\pi}\Bigg(\frac{1}{2}(1+\cos^2\Theta) + \frac{1}{2}(1-3\cos^2\Theta)\rho_{00} + \sqrt{2}Re\rho_{10}\sin(2\Theta)\cos\Phi + \rho_{1-1}\sin^2\Theta\cos 2\Phi\Bigg)\,.$$

$\cos\Theta, \Phi$ **angles of photon from** $\omega$ **decay in the** $\omega$ **rest frame**

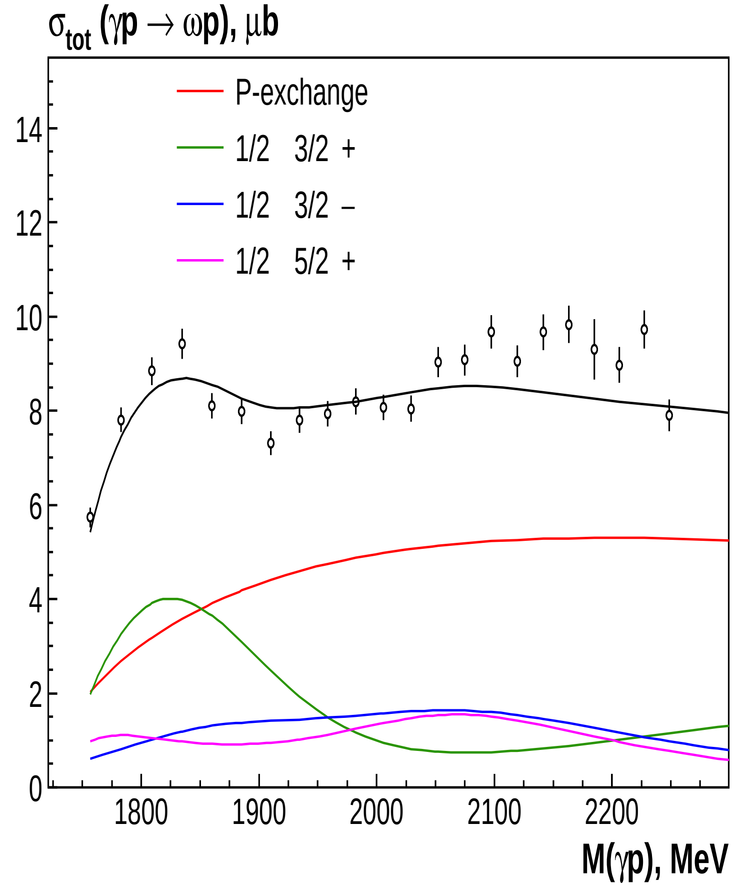
# Fit of the density matrices $\gamma p \to p\omega$ (CB-ELSA) (A.Wilson)

$\rho^0_{00}$ $\rho^0_{1-1}$



# Photoproduction of vector mesons. $\gamma p \to p\omega$ (A.Wilson)




- **Strong contribution from the $P_{13}$ partial wave: interference of $P_{13}(1700)$ and $P_{13}(1900)$ states.**
- **A confirmation of the $F_{15}(2000)$ state.**
- **A structure in the $D_{13}$ partial wave in the region 2100 MeV.**
- **No large contributions either from $7/2^+$ or $7/2^-$ states are found**