

Chiral Symmetry and Low-Energy Pion-Photon Reactions

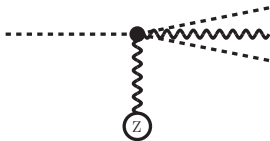
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- Tests of **Chiral Perturbation Theory** via low-energy $\pi^- \gamma$ reactions
- COMPASS@CERN: Primakoff effect to extract $\pi^- \gamma$ cross sections
- π -Compton scattering $\pi^- \gamma \rightarrow \pi^- \gamma$: electric/magnetic polarizabilities
- Radiative corrections to π -Compton scattering (and $\mu^\pm p \rightarrow \mu^\pm p$)
- Chiral anomaly test: $\pi^- \gamma \rightarrow \pi^- \pi^0$
- Neutral and charged **pion-pair production**: $\pi^- \gamma \rightarrow \pi^- \pi^0 \pi^0$, $\pi^+ \pi^- \pi^-$
- Radiative corrections to $\pi^- \gamma \rightarrow 3\pi$
- Radiative corrections to proton and neutron magnetic moments

COMPASS experiment at CERN (S. Paul, J. Friedrich, B. Ketzer, B. Grube,...)

Primakoff effect:



- Scattering high-energy pions in nuclear Coulomb field (charge Z) allows to extract cross sections for $\pi^- \gamma$ reactions (equivalent-photon method)

$$\frac{d\sigma}{ds dQ^2} = \frac{Z^2 \alpha}{\pi(s - m_\pi^2)} \frac{Q^2 - Q_{min}^2}{Q^4} \sigma_{\pi^- \gamma}(s), \quad Q_{min} = \frac{s - m_\pi^2}{2E_{beam}}$$

- $s = (\pi^- \gamma \text{ invariant mass})^2$, $Q \rightarrow 0$ momentum transfer by virtual photon
- Isolate Coulomb peak from strong interaction background
- Different final-states $\pi^- \gamma$, $\pi^- \pi^0$, $\pi^- \pi^0 \pi^0$, $\pi^+ \pi^- \pi^-$ allow to test different aspects of chiral dynamics (low-energy QCD)
- Diffractive pion-scattering: meson spectroscopy and search for exotics

Structure of pion at low energies: calculated in chiral perturbation theory

- General form of pion Compton-scattering amplitude in cm frame:

$$T_{\pi\gamma} = 8\pi\alpha \left\{ -\vec{\epsilon}_1 \cdot \vec{\epsilon}_2 A(s, t) + \vec{\epsilon}_1 \cdot \vec{k}_2 \vec{\epsilon}_2 \cdot \vec{k}_1 \frac{2}{t} [A(s, t) + B(s, t)] \right\}, \quad t = (k_1 - k_2)^2$$

- Corresponding differential cross section:

$$\frac{d\sigma}{d\Omega_{cm}} = \frac{\alpha^2}{2s} \left\{ |A(s, t)|^2 + |A(s, t) + (1 + \cos\theta_{cm})B(s, t)|^2 \right\}$$

- Tree diagrams (s-channel pole diagram vanishes, $\epsilon_1 \cdot (2p_1 + k_1) = 0$):

$$A(s, t) = 1, \quad B(s, t) = \frac{s - m_\pi^2}{m_\pi^2 - s - t}$$

- One-loop diagrams (finite after mass renormalization):

$$A(s, t) = -\frac{1}{(4\pi f_\pi)^2} \left\{ \frac{t}{2} + 2m_\pi^2 \ln^2 \frac{\sqrt{4m_\pi^2 - t} + \sqrt{-t}}{2m_\pi} \right\} \sim t^2 > 0$$

- Electric/magnetic polarizabilities = low-energy const. with $\alpha_\pi + \beta_\pi = 0$

$$A(s, t) = -\frac{\beta_\pi m_\pi t}{2\alpha} < 0, \quad \alpha_\pi - \beta_\pi = \frac{\alpha(\bar{l}_6 - \bar{l}_5)}{24\pi^2 f_\pi^2 m_\pi}$$

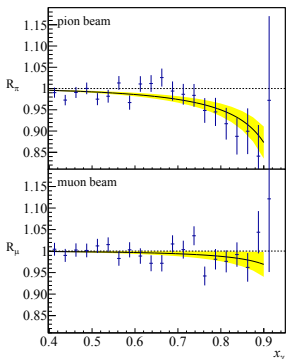
- Combination $\bar{l}_6 - \bar{l}_5 = 3.0 \pm 0.3$ determined via radiative pion decay
 $\pi^+ \rightarrow e^+ \nu_e \gamma$, PIBETA@PSI: axial-to-vector coupl. ratio $F_A/F_V = 0.44$

- Two-loop prediction of chiral perturbation theory [J. Gasser et al. ('06)]

$$\alpha_\pi - \beta_\pi = \frac{\alpha(\bar{\ell}_6 - \bar{\ell}_5)}{24\pi^2 f_\pi^2 m_\pi} + \frac{\alpha m_\pi}{(4\pi f_\pi)^4} \left\{ c^r + \frac{8}{3} (\bar{\ell}_2 - \bar{\ell}_1 + \bar{\ell}_5 - \bar{\ell}_6 + \frac{65}{12}) \ln \frac{m_\pi}{m_\rho} + \frac{4}{9} (\bar{\ell}_1 + \bar{\ell}_2) - \frac{\bar{\ell}_3}{3} + \frac{4\bar{\ell}_4}{3} (\bar{\ell}_6 - \bar{\ell}_5) - \frac{187}{81} + \left(\frac{53\pi^2}{48} - \frac{41}{324} \right) \right\}$$

$$\alpha_\pi - \beta_\pi = (5.7 \pm 1.0) \cdot 10^{-4} \text{ fm}^3, \quad \alpha_\pi + \beta_\pi = 0.16 \cdot 10^{-4} \text{ fm}^3$$

- COMPASS result: $\alpha_\pi - \beta_\pi = (4.0 \pm 1.8) \cdot 10^{-4} \text{ fm}^3$ [PRL 114, 062002 ('15)]



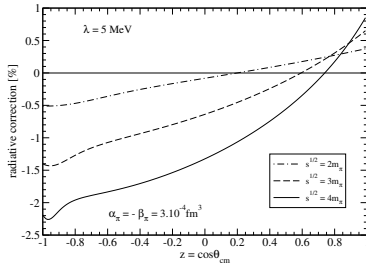
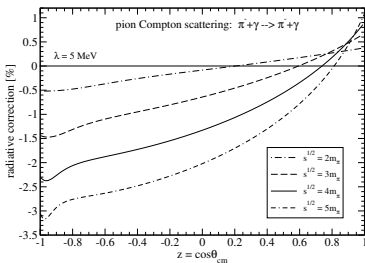
$$x_\gamma = E_\gamma / E_\pi \text{ in lab, } \cos \theta_{\text{cm}} = 1 - 2x_\gamma s / (s - m_\pi^2)$$

Analysis of data includes:

- chiral pion-loop corrections $A(s, t) \sim \ln^2(.t.)$
- radiative corrections [NPA 812, 186 ('08)]
- isospin-breaking correction $\sim (m_\pi^2 - m_{\pi^0}^2) \ln^2(.t.)$
- previous results from Mainz and Serpukhov:
 $\alpha_\pi - \beta_\pi = (12 - 16) \cdot 10^{-4} \text{ fm}^3$

Radiative corrections to pion Compton scattering

- Pion-structure effects small: necessary to include radiative corr. of $\mathcal{O}(\alpha)$
- Start with structureless pion: extensive exercise in one-loop scalar QED
- Include leading pion-structure $\alpha_\pi - \beta_\pi$ in form of $\gamma\gamma$ -contact vertex $F_{\mu\nu}F^{\mu\nu}$
- Virtual photon loops + soft γ -radiation ($\omega < \lambda$) give infrared finite result



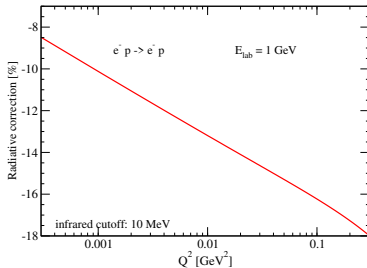
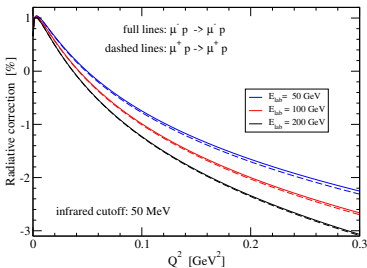
- QED radiative corrections are maximal in backward directions $z \simeq -1$
- Same kinematical signature as pion polarizability difference $\alpha_\pi - \beta_\pi$
- Suppressed by factor of ~ 10
- Relative size and angular depend. not affected by leading pion-structure

Proton radius from elastic muon-proton scattering

- COMPASS proposal [S. Paul, J. Friedrich, et al.]:
Measure proton charge radius $r_p = (0.84 - 0.88)$ fm in $\mu^\pm p \rightarrow \mu^\pm p$ scatter.
- Generalize Rosenbluth formula to massive muons ($m_\mu \geq \sqrt{-t} = Q$)

$$\frac{d\sigma^{(1\gamma)}}{dt} = \frac{4\pi\alpha^2}{t^2} [s - (M_p + m_\mu)^2]^{-1} [s - (M_p - m_\mu)^2]^{-1} \\ \times \left\{ \left[\frac{(s + M_p^2 - m_\mu^2)^2}{4M_p^2 - t} + m_\mu^2 - s \right] \left[4M_p^2 G_E^2(t) - t G_M^2(t) \right] + t \left(m_\mu^2 + \frac{t}{2} \right) G_M^2(t) \right\}$$

- Advantage of muons over electrons: much smaller radiative corrections



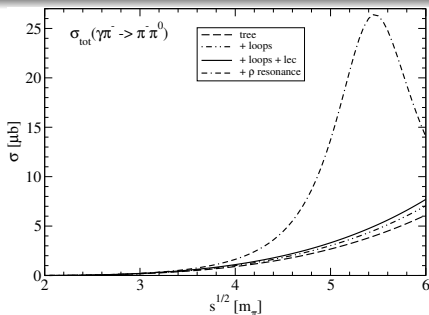
- Analytical calculation of radiative corrections done for point-like proton:
Vertex corrections, vacuum polarization, 2-photon exchange, soft bremsstrahlung

Extracting the chiral anomaly

- $\pi^0 \rightarrow 2\gamma$ and $\gamma \rightarrow 3\pi$ couplings determined by chiral anomaly of QCD
- Amplitude and cross section for $\pi^-(p_1) + \gamma(k, \epsilon) \rightarrow \pi^-(p_2) + \pi^0(p_0)$:

$$T_{\gamma 3\pi} = \frac{e}{4\pi^2 f_\pi^3} \epsilon_{\mu\nu\kappa\lambda} \epsilon^\mu p_1^\nu p_2^\kappa p_0^\lambda M(s, t), \quad F_{3\pi} = 9.8 \text{ GeV}^{-3}$$

$$\sigma_{\text{tot}}(s) = \frac{\alpha(s - m_\pi^2)(s - 4m_\pi^2)^{3/2}}{(4f_\pi)^6 \pi^4 \sqrt{s}} \int_{-1}^1 dz (1 - z^2) |M(s, t)|^2$$



- $\rho(770)$ -resonance must be included:

$$M(s, t)^{(\rho)} = 1 + 0.46 \left\{ \frac{s}{m_\rho^2 - s - i\sqrt{s}\Gamma_\rho(s)} + \frac{t}{m_\rho^2 - t} + \frac{u}{m_\rho^2 - u} \right\}$$

Extracting the chiral anomaly

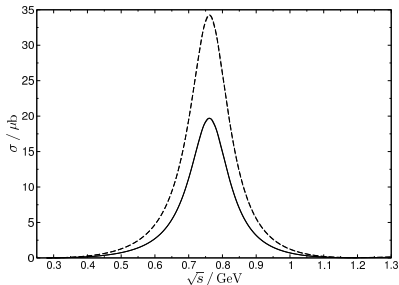
- Dispersive representation of $\pi\gamma \rightarrow \pi\pi$ with p-wave phase shift as input
[M. Hoferichter, B. Kubis, D. Sakkas, PRD 86, 116009 ('12)]

$$\frac{e}{4\pi^2 f_\pi^3} M(s, t) = F(s) + F(t) + F(u), \quad u = 3m_\pi^2 - s - t,$$

$$F(s) = a + bs + \frac{s^2}{\pi} \int_{4m_\pi^2}^{\infty} ds' \frac{\text{Im}F(s')}{s'^2(s'-s)}, \quad \text{Im}F(s) = [F(s) + \hat{F}(s)] \sin \delta_1^1(s) e^{-i\delta_1^1(s)}$$

- Relevant subtraction constant $C = 3(a + b m_\pi^2)$ is fitted to data and matched via the chiral representation to $F_{3\pi}$

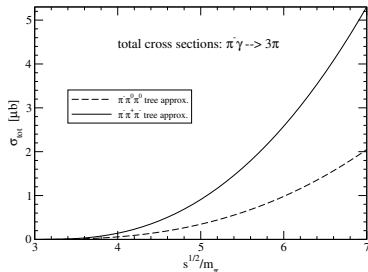
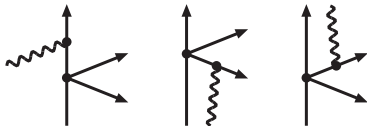
$$C = F_{3\pi} \left\{ 1 + \frac{m_\pi^2}{(4\pi f_\pi)^2} \left(2.9 - \ln \frac{m_\pi}{m_\rho} \right) \right\} = 1.067 F_{3\pi}$$



- solid line: $C = 9.78 \text{ GeV}^{-3}$
dashed line: $C = 12.9 \text{ GeV}^{-3}$
- close to threshold, one-photon exchange an important correction:
 $1 \rightarrow 1 - 2e^2 f_\pi^2 / t$
- Good theory waiting for good data

Tree level cross sections for $\pi^- \gamma \rightarrow 3\pi$

- Coulomb gauge $\epsilon \cdot p_1 = \epsilon \cdot k = 0$, photon does not couple to incoming π^-
- No $\gamma 4\pi$ vertex at leading order



- Example: total cross section for $\pi^-(p_1) + \gamma(k, \epsilon) \rightarrow \pi^- \pi^0 \pi^0$

$$\sigma_{tot}(s) = \frac{\alpha}{16\pi^2 f_\pi^4 (s - m_\pi^2)^3} \int_{2m_\pi}^{\sqrt{s} - m_\pi} d\mu \sqrt{\mu^2 - 4m_\pi^2} (\mu^2 - m_\pi^2)^2 \left\{ (s + m_\pi^2 - \mu^2) \ln \frac{s + m_\pi^2 - \mu^2 + \lambda^{1/2}(s, \mu^2, m_\pi^2)}{2m_\pi \sqrt{s}} - \lambda^{1/2}(s, \mu^2, m_\pi^2) \right\}$$

- $(\mu^2 - m_\pi^2)/f_\pi^2$ is LO chiral $\pi\pi$ -interaction, rest from 3-body phase space
- How large are next-to-leading order corrections from chiral loops + cts?

- 3-body process: $\pi^-(p_1) + \gamma(k, \epsilon) \rightarrow \pi^-(p_2) + \pi^0(q_1) + \pi^0(q_2)$
- General form of T-matrix (in Coulomb gauge)

$$T_{\gamma 4\pi} = \frac{2e}{f_\pi^2} \left[\vec{\epsilon} \cdot \vec{q}_1 A_1 + \vec{\epsilon} \cdot \vec{q}_2 A_2 \right], \quad A_2 = A_1 | (s_1 \leftrightarrow s_2, t_1 \leftrightarrow t_2)$$

- Amplitudes A_1 and A_2 depend on five (independ.) Mandelstam variables:

$$s = (p_1 + k)^2, \quad s_1 = (p_2 + q_1)^2, \quad s_2 = (p_2 + q_2)^2, \quad t_1 = (q_1 - k)^2, \quad t_2 = (q_2 - k)^2$$

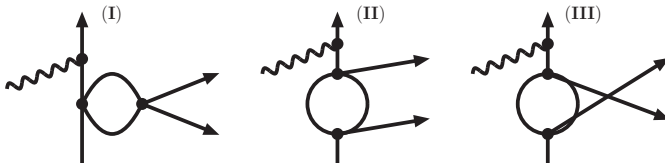
- Convenient for permutation of identical neutral pions ($s_1 \leftrightarrow s_2, t_1 \leftrightarrow t_2$)
- Tree-level amplitudes:

$$A_1^{(\text{tree})} = A_2^{(\text{tree})} = \frac{2m_\pi^2 + s - s_1 - s_2}{3m_\pi^2 - s - t_1 - t_2}$$

Chiral corrections: [N. Kaiser, NPA 848, 198 ('10)]

Radiative correct.: [EPJA 46, 373 ('10); N.K. + S. Petschauer, EPJA 49, 159 ('13)]

- Pion-loop corrections (example I)

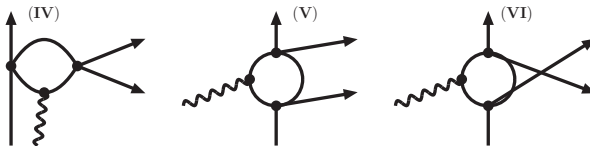


$$A_1^{(I)} = \frac{1}{(4\pi f_\pi)^2} \frac{2m_\pi^2 + s - s_1 - s_2}{3m_\pi^2 - s - t_1 - t_2} \left\{ \left(\xi + \ln \frac{m_\pi}{\mu} \right) (s_1 + s_2 + t_1 + t_2 - 11m_\pi^2) \right. \\ \left. + (s_1 + s_2 + t_1 + t_2 - 7m_\pi^2) \left[J(3m_\pi^2 + s - s_1 - s_2) - \frac{1}{2} \right] \right\}$$

- Loop function (from loop with two pion-propagators)

$$J(s) = \sqrt{\frac{s - 4m_\pi^2}{s}} \left[\ln \frac{\sqrt{|s - 4m_\pi^2|} + \sqrt{|s|}}{2m_\pi} - \frac{i\pi}{2} \theta(s - 4m_\pi^2) \right], \quad s < 0 \text{ or } s > 4m_\pi^2$$

- Pion-loop corrections (example IV)

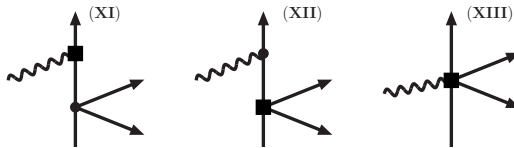


$$\begin{aligned}
 A_1^{(IV)} = & \frac{2m_\pi^2 + s - s_1 - s_2}{(4\pi f_\pi)^2} \left\{ \xi + \ln \frac{m_\pi}{\mu} - \frac{1}{2} + J(3m_\pi^2 + s - s_1 - s_2) \right. \\
 & + \frac{m_\pi^2 - s}{2m_\pi^2 - t_1 - t_2} + \frac{2(s - m_\pi^2)}{(2m_\pi^2 - t_1 - t_2)^2} \left\{ (s_1 + s_2 - s - m_\pi^2 - t_1 - t_2) \right. \\
 & \times \left[J(m_\pi^2 + s - s_1 - s_2 + t_1 + t_2) - J(3m_\pi^2 + s - s_1 - s_2) \right] \\
 & \left. \left. + 2m_\pi^2 \left[G(m_\pi^2 + s - s_1 - s_2 + t_1 + t_2) - G(3m_\pi^2 + s - s_1 - s_2) \right] \right\} \right\}
 \end{aligned}$$

- Loop function (from loop with three pion-propagators)

$$G(s) = \left[\ln \frac{\sqrt{|s - 4m_\pi^2|} + \sqrt{|s|}}{2m_\pi} - \frac{i\pi}{2} \theta(s - 4m_\pi^2) \right]^2, \quad s < 0 \text{ or } s > 4m_\pi^2$$

- Corrections from chiral counterterms: higher-dimensional operators, incorporate unresolved short-distance dynamics of Goldstone bosons



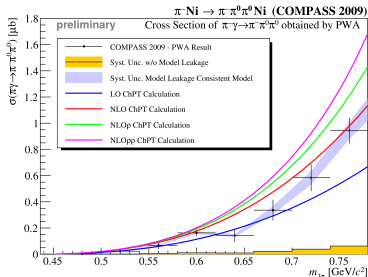
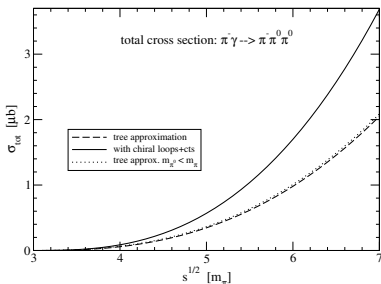
- Complete counterterm contribution:

$$A_1^{(\text{ct})} = \frac{1}{(4\pi f_\pi)^2} \frac{1}{3m_\pi^2 - s - t_1 - t_2} \left\{ \frac{\bar{\ell}_1}{3} (s_1 + s_2 - s - m_\pi^2)^2 + \frac{\bar{\ell}_2}{3} [s^2 + s_1^2 + s_2^2 + t_2^2 - 2ss_1 + (s - 2s_1 + 2s_2 - t_1)t_2 + m_\pi^2(s - 6s_2 + t_1 - 2t_2 + 6m_\pi^2)] - \frac{\bar{\ell}_3}{2} m_\pi^4 + 2\bar{\ell}_4 m_\pi^2 (s + 2m_\pi^2 - s_1 - s_2) \right\}$$

- Values of low-energy constants: $\bar{\ell}_1 = -0.4 \pm 0.6$, $\bar{\ell}_2 = 4.3 \pm 0.1$, $\bar{\ell}_3 = 2.9 \pm 2.4$, $\bar{\ell}_4 = 4.4 \pm 0.2$, determined with improved empirical input
- Uncertainty induced by errorbars of $\bar{\ell}_j$: about $\pm 5\%$ for $\sigma_{\text{tot}}(s)$, mainly $\bar{\ell}_1$
- Chiral resonance amplitudes for $\gamma^* \rightarrow 4\pi$ [Ecker+Unterdorfer, EPJC 24, 535 ('02)]

- Total cross section for $\pi^- \gamma \rightarrow 3\pi$

$$\sigma_{\text{tot}}(s) = \frac{\alpha}{32\pi^3 f_\pi^4 (s - m_\pi^2)} \iint_{z^2 < 1} d\omega_1 d\omega_2 \int_{-1}^1 dx \int_0^\pi d\phi \left| \hat{k} \times (\vec{q}_1 A_1 + \vec{q}_2 A_2) \right|^2$$

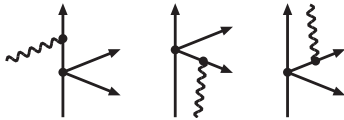


- Enhancement of $\sigma_{\text{tot}}(s)$ by factor 1.5 - 1.8 through chiral corrections
- Suggestive explanation: $\pi^+ \pi^- \rightarrow \pi^0 \pi^0$ final state interaction $(1 + 0.20)^2$

$$\frac{1}{3}(a_0 - a_2) = \frac{3m_\pi}{32\pi f_\pi^2} \left[1 + \frac{m_\pi^2}{36\pi^2 f_\pi^2} \left(\bar{\ell}_1 + 2\bar{\ell}_2 - \frac{3\bar{\ell}_3}{8} + \frac{9\bar{\ell}_4}{2} + \frac{33}{8} \right) \right]$$

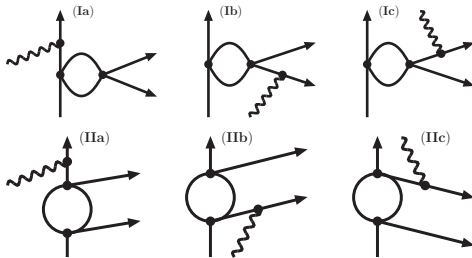
Charged pion-pair production

- 3-body process: $\pi^-(p_1) + \gamma(k, \epsilon) \rightarrow \pi^+(p_2) + \pi^-(q_1) + \pi^-(q_2)$
- Photon couples to all charged pions: \rightarrow many more diagrams

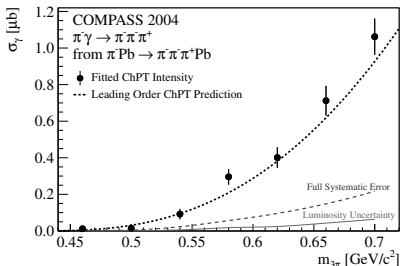
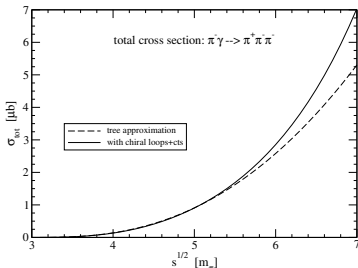


$$A_1^{(\text{tree})} = \frac{2s - 2m_\pi^2 - s_1 - s_2 + t_1 + t_2}{3m_\pi^2 - s - t_1 - t_2} + \frac{s - s_1 - s_2 + t_2}{t_1 - m_\pi^2}$$

$$A_2^{(\text{tree})} = \frac{2s - 2m_\pi^2 - s_1 - s_2 + t_1 + t_2}{3m_\pi^2 - s - t_1 - t_2} + \frac{s - s_1 - s_2 + t_1}{t_2 - m_\pi^2}$$



- Total cross section for $\pi^- \gamma \rightarrow \pi^+ \pi^- \pi^-$



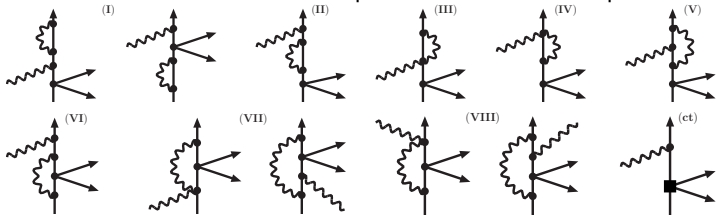
- $\sigma_{\text{tot}}(s)$ for $\sqrt{s} < 6m_\pi$ almost unchanged in comparison to tree approx.
- Suggestive explanation: $\pi^- \pi^- \rightarrow \pi^- \pi^-$ final state interaction $(1 - 0.02)^2$

$$a_2 = -\frac{m_\pi}{16\pi f_\pi^2} \left[1 - \frac{m_\pi^2}{12\pi^2 f_\pi^2} \left(\bar{\ell}_1 + 2\bar{\ell}_2 - \frac{3\bar{\ell}_3}{8} - \frac{3\bar{\ell}_4}{2} + \frac{3}{8} \right) \right]$$

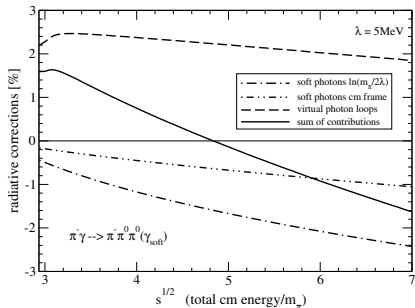
- Analysis of COMPASS data for $\sqrt{s} \leq 5m_\pi$ agrees with ChPT prediction
 First measurement of chiral dynamics in $\pi^- \gamma \rightarrow \pi^- \pi^- \pi^+$, PRL108, 192001 ('12)
- Agreement on level of full 5-dimensional phase space distribution

Radiative corrections to neutral pion-pair production

- Chiral $\pi^+\pi^- \rightarrow \pi^0\pi^0$ transition amplitude factors out of all photon loops



- Observe: Only "irreducible" photon-loop diagrams in 2nd row contribute

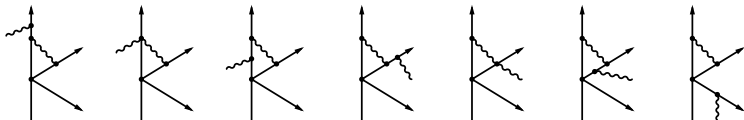


- Radiative correction to total cross section varies between +2% and -2%

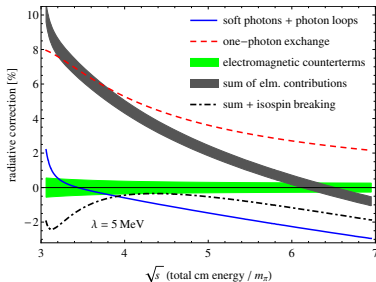
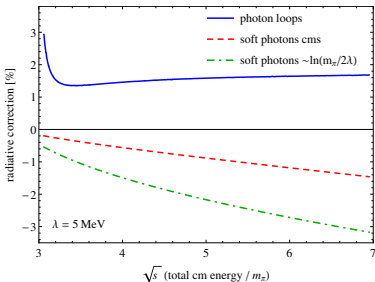


Radiative corrections to charged pion-pair production

- 42 irreducible photon-loop diagrams: virtual photon connects two differ. charged pions, all couplings of incoming photon [FeynCalc + LoopTools]



- Radiative corrections to total cross section $\sigma_{\text{tot}}(\pi^- \gamma \rightarrow \pi^+ \pi^- \pi^-)$



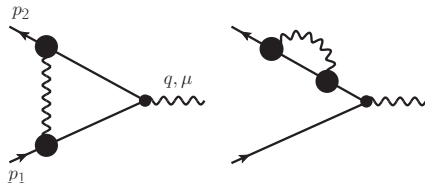
- Photon-loops almost constant, near threshold Coulomb singularity α/v
- Coulomb singularity causes kink in $\pi^+ \pi^-$ and $\pi^- \pi^-$ mass spectra
- One-photon exchange is sizeable, compensated partly by leading isospin-breaking correction $\delta_{ib} = 2(m_{\pi^0}/m_{\pi})^2 - 2 = -0.13$

Radiative corrections to magnetic moments of proton and neutron

- Measured values of proton/neutron magnetic moment extremely precise

$$\mu_p = 1 + \kappa_p = 2.7928473 \quad \mu_n = 0 + \kappa_n = -1.913042, \quad \text{in units of } \frac{e\hbar}{2M_p}$$

- Large anomalous magnetic moments κ_p, κ_n arise from strong interaction, but at given expt. precision electromagnetic effects play also a role: 10^{-3}
- Adapt Schwinger calc. for leptons $\delta\kappa_l = \frac{\alpha}{2\pi}$ to nucleons with structure $F_{1,2}$



- On-shell vector-vertex: $\gamma^\mu F_1^{(\gamma)}(t) + \frac{i}{2M} \sigma^{\mu\nu} q_\nu F_2^{(\gamma)}(t)$ with $t = (p_2 - p_1)^2 \leq 0$
- Project out $F_2^{(\gamma)}(t)$, careful limit $t \rightarrow 0$, Wick-rotation, angular integration

$$\int \frac{d^4l}{(2\pi)^4 i} \frac{1}{(-l^2)} [\dots] = \frac{M^2}{4\pi^3} \int_0^\infty dx \int_{-1}^1 dz x \sqrt{1-z^2} [\dots]$$

- Inclusion of wavefunction renormalization factor Z_2 from self-energy sub-diagram leads to **infrared-finite** and **gauge-invariant** result

Radiative corrections to magnetic moments of proton and neutron

- Radiatively induced magnetic moment: $c = 1, 0$ and $\kappa = 1.793, -1.913$

$$\delta\kappa = \frac{\alpha}{\pi} \int_0^\infty dx \left\{ \left[\left(\frac{2 + 4x^2 + x^4}{\sqrt{4 + x^2}} - 2x - x^3 \right) c + \left(\frac{x^2(10 + 3x^2)}{\sqrt{4 + x^2}} - 4x - 3x^3 \right) \frac{\kappa}{4} \right] F_1^2(xM) \right.$$

$$+ \left[\left(\frac{x^2(7 + 2x^2)}{\sqrt{4 + x^2}} - 3x - 2x^3 \right) c + \left(\frac{x^2(10 + 3x^2)}{\sqrt{4 + x^2}} - 4x - 3x^3 \right) \frac{\kappa}{2} \right] F_1(xM)F_2(xM)$$

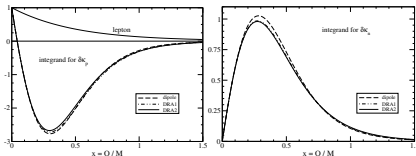
$$+ \left[\left(\frac{x^2}{\sqrt{4 + x^2}} (16 + x^2 - x^4) - 8x - 3x^3 + x^5 \right) \frac{c}{8} \right.$$

$$\left. + \left(\frac{x^2}{\sqrt{4 + x^2}} (64 + 16x^2 - x^4) - 32x - 18x^3 + x^5 \right) \frac{\kappa}{32} \right] F_2^2(xM) \left. \right\}$$

- Numerical values of $\delta\kappa$ and corresponding integrands (without α/π)

form factor	dipole	DRA1	DRA2
$10^3 \cdot \delta\kappa_p$	-3.47	-3.49	-3.42
$10^3 \cdot \delta\kappa_n$	1.37	1.34	1.34

dispersion relation analyses



- Photon-loops with internal $\Delta(1232)$ -isobars: Rarita-Schwinger formalism
- Convent. $\Delta N\gamma$ -vertex + propagator: $\delta\kappa_p = -0.9 \cdot 10^{-3}$, $\delta\kappa_n = 1.2 \cdot 10^{-3}$
- Using spin-3/2 projected versions: $\delta\kappa_p = 0.0 \cdot 10^{-3}$, $\delta\kappa_n = -0.8 \cdot 10^{-3}$
- Estimate of genuine strong parts: $\kappa_p^{(strong)} = 1.797$, $\kappa_n^{(strong)} = -1.915$