

# Recipe to construct the amplitude for 2-particle decays in tensor formalism,v2

Helmut Koch

May 5, 2014

## Abstract

A recipe is given how to construct the decay amplitude for  $1 \rightarrow 2 + 3$ ;  $2 \rightarrow 4 + 5$ ;  $3 \rightarrow 6 + 7$  in tensor formalism (Rarita-Schwinger) for arbitrary integral spins.

## 1 Introduction

Increasingly, modern PWA-analyses use the full relativistic invariant tensor formalism (Rarita-Schwinger) [1, 2, 7, 4, 5, 6, 11] to express the amplitudes. The reason is obvious: Standard methods (Helicity-, Zemach-formalisms) disregard the relativistic effects connected with the speed of resonances. In case of low speed, the effect is negligible, but in many cases essential effects occur [7]. In addition, the tensor formalism ensures the correct behaviour of the partial wave amplitudes near thresholds. An introduction into the formalism is given in [1, 4, 7, 8, 9, 10], based on [13, 15, 16, 12, 14].

## 2 Amplitude for $1 \rightarrow 2 + 3$

The amplitude is built up of the spin wave functions, the angular momentum tensor and the spin projection tensor. All these tensors are defined in [1, 3, 4, 7, 9, 10], but their basic features are briefly discussed in the following.

Spin wave functions (polarization vectors):  $\epsilon^{(s)}(p, m_s)$

$p$  is the four momentum of the particle with spin  $s, m_s$  its spin projection. The spin wave functions are called pure spin tensors and represent quantities of the dimension  $2J+1$ . They are symmetric, traceless and are orthogonal to the four-momentum as given in the parenthesis. They obey the Rarita-Schwinger

conditions:

$$\epsilon_{\mu_1\mu_2\cdots\mu_i\cdots\mu_j\cdots\mu_s}^{(s)} = \epsilon_{\mu_1\mu_2\cdots\mu_j\cdots\mu_i\cdots\mu_s}^{(s)}$$

$$g^{\mu_i\mu_j}\epsilon_{\mu_1\mu_2\cdots\mu_i\cdots\mu_j\cdots\mu_s}^{(s)} = 0$$

$$p^{\mu_i}\epsilon_{\mu_1\mu_2\cdots\mu_i\cdots\mu_s}^{(s)} = 0 \quad (\text{see [7, 9, 10, 13]})$$

For a massive spin 1 particle with mass M the spin wave function is [7]

$$\epsilon^{(1)\mu}(p, m_s = \mp 1) = \frac{\pm 1}{M\sqrt{2}} \begin{pmatrix} p_x \mp ip_y \\ M + p_x(p_x \mp ip_y)/(E + M) \\ \mp iM + p_y(p_x \mp ip_y)/(E + M) \\ p_z(p_x \mp ip_y)/(E + M) \end{pmatrix},$$

$$\epsilon^{(1)\mu}(p, m_s = 0) = \frac{1}{M} \begin{pmatrix} p_z \\ p_z p_x/(E + M) \\ p_z p_y/(E + M) \\ M + p_z^2/(E + M) \end{pmatrix}$$

The spin wave function for a massive spin 2 particle is given by:

$$\epsilon_{\mu\nu}^{(2)}(p, m) = \sum_{m_1, m_2} \langle 1, m_1, 1, m_2 | 2, m \rangle \epsilon_{\mu}^{(1)}(p, m_1) \epsilon_{\nu}^{(1)}(p, m_2)$$

...

Orbital momentum tensors:  $\tilde{t}^{(l)}(p_1)$

They are symmetric, traceless and are real objects. For a decay  $1 \rightarrow 2 + 3$  they are orthogonal to the four momentum  $p_1 = p_2 + p_3$  of the mother particle and obey the Rarita-Schwinger conditions. They are built up of the relative four momenta  $r_{23} = p_2 - p_3$ . The tilde means, that they are pure spin tensors. Examples are:

$$\tilde{t}^{(0)}(p_1) = 1$$

$$\tilde{t}_{\mu}^{(1)}(p_1) = \tilde{g}_{\mu\nu}(p_1)r^{\nu} \equiv \tilde{r}_{\mu}(p_1)$$

$$\tilde{t}_{\mu\nu}^{(2)}(p_1) = \tilde{r}_{\mu}\tilde{r}_{\nu} - 1/3(\tilde{r} \cdot \tilde{r})\tilde{g}_{\mu\nu}(p_1) \quad \text{with } \tilde{g}_{\mu\nu}(p_1) = g_{\mu\nu} - \frac{p_{1\mu}p_{1\nu}}{p_1^2}$$

...

They have the  $p^l$  dependence near threshold as required for partial waves.

Spin projection tensors  $P^{(s)}(p)$ :

$P_{\alpha_1\cdots\alpha_s\alpha'_1\cdots\alpha'_s}^{(s)}(p)$  is the spin projection tensor. It has  $2s$  indices (rank  $2s$ ). Applied to arbitrary tensors it ensures, that the resulting tensor verifies the Rarita-Schwinger conditions. The resulting tensor is symmetrized.

It is built up of the invariant tensors of the Poincare-group, i.e.

$$g^{\mu\nu} = g^{\alpha\beta} \Lambda_{\alpha}^{\mu} \Lambda_{\beta}^{\nu} \quad \text{and} \quad \varepsilon^{\nu\rho\sigma\tau} = \varepsilon^{\mu\alpha\beta\gamma} \Lambda_{\mu}^{\nu} \Lambda_{\alpha}^{\rho} \Lambda_{\beta}^{\sigma} \Lambda_{\gamma}^{\tau} \quad (\varepsilon = \text{Levi-Civita tensor})$$

Spin projection tensors occur in two different situations:

(1) when two amplitudes are multiplied

(2) when two spins are added to a total spin

Examples are:

$$P_{\alpha_1\alpha'_1}^{(1)}(p) = \sum_m \epsilon_{\alpha_1}^{(1)}(p, m) \epsilon_{\alpha'_1}^{(1)*}(p, m) = -g_{\alpha_1\alpha'_1} + \frac{p_{\alpha_1}p_{\alpha'_1}}{p^2} \equiv -\tilde{g}_{\alpha_1\alpha'_1}(p)$$

$$P_{\alpha_1\alpha_2\alpha'_1\alpha'_2}^{(2)}(p) = \sum \epsilon_{\alpha_1\alpha_2}^{(2)}(p, m) \epsilon_{\alpha'_1\alpha'_2}^{(2)*}(p, m) = 1/2(\tilde{g}_{\alpha_1\alpha'_1}(p)\tilde{g}_{\alpha_2\alpha'_2}(p) + \tilde{g}_{\alpha_1\alpha'_2}(p)\tilde{g}_{\alpha_2\alpha'_1}(p)) - 1/3\tilde{g}_{\alpha_1\alpha_2}(p)\tilde{g}_{\alpha'_1\alpha'_2}(p)$$

...

In particular:

$$P_{\alpha_1\dots\alpha_s\alpha'_1\dots\alpha'_s}^{(s)}(p) \epsilon^{(s)\alpha'_1\dots\alpha'_s}(p) = \epsilon^{(s)\alpha_1\dots\alpha_s}(p) \text{ and } \epsilon^{(s)\alpha'_1\dots\alpha'_s}(p) P_{\alpha_1\dots\alpha_s\alpha'_1\dots\alpha'_s}^{(s)}(p) = \epsilon_{\alpha_1\dots\alpha_s}^{(s)}(p)$$

$$P_{\alpha_1\dots\alpha_n\alpha'_1\dots\alpha'_n}^{(n)}(p) \tilde{t}^{(n)\alpha'_1\dots\alpha'_n}(p) = \tilde{t}^{(n)\alpha_1\dots\alpha_n}(p) \text{ and } \tilde{t}^{(n)\alpha'_1\dots\alpha'_n}(p) P_{\alpha_1\dots\alpha_n\alpha'_1\dots\alpha'_n}^{(n)}(p) = \tilde{t}_{\alpha_1\dots\alpha_n}^{(n)}(p)$$

These relations are only valid in application of P to a pure spin tensor when both have equal momenta. Here  $\tilde{g}^{\mu\nu} = g^{\mu\nu}$ . For the proof the symmetry of the tensors is used. The relations are also valid, when the rank of  $\epsilon, \tilde{t}$  is larger than half the rank of P.

They are not valid for cases like  $P_{\alpha_1\alpha_2\alpha_3\alpha'_1\alpha'_2\alpha'_3}^{(3)} \tilde{t}^{(2)\alpha'_1\alpha'_2}$  or  $P_{\alpha_1\alpha_2\alpha_3\alpha'_1\alpha'_2\alpha'_3}^{(3)} \tilde{t}_{\alpha'_3}^{(3)\alpha'_1\alpha'_2}$

Now lets come back to the amplitude built on tensor formalism.

Notation:  $1(s_1, m_1) \rightarrow 2(s_2, m_2) + 3(s_3, m_3)$

Four momenta:  $p_1, p_2, p_3; p_1 = p_2 + p_3$

with s=spin; m=spin projection;  $l_{23}$  = relative angular momentum;  $s_{23}$  = total spin of particles 2 and 3

LS-scheme:  $|s_2 - s_3| \leq s_{23} \leq s_2 + s_3; l_{23}$  satisfies  $s_1 = s_{23} \oplus l_{23}$

The amplitude for given  $s_1, s_2, s_3, m_1, m_2, m_3, s_{23}$  and  $l_{23}$  has the structure([1, 4]):

$$A(1_{m_1} \rightarrow 2_{m_2} + 3_{m_3}) = \Lambda_{l_{23}s_{23}} \epsilon_{\mu_1\mu_2\dots}^{(s_1)}(p_1, m_1) [\dots] \tilde{t}_{\dots}^{(l_{23})}(p_1) \chi_{\nu_1\nu_2\dots}^{(s_{23})}(p_1)$$

( $\Lambda$  = coupling constant)

with

[..] = 1 for  $s_{23} + l_{23} + s_1 = \text{even}$

[..] =  $\epsilon^{\mu\alpha\beta\gamma} p_{1\gamma}$  for  $s_{23} + l_{23} + s_1 = \text{odd}$

A projection tensor  $P^{(s_1)}(p_1)$  could be added before [..], but is redundant here (see above).

and with the spin tensor of particles 2,3

$$\chi_{\nu_1\nu_2\dots}^{(s_{23})}(p_1) = P_{\dots}^{(s_{23})}(p_1) [\dots] \epsilon_{\dots}^{*(s_2)}(p_2, m_2) \epsilon_{\dots}^{*(s_3)}(p_3, m_3)$$

with

[..]=1 for  $s_{23} + s_2 + s_3 = \text{even}$

[..]= $\epsilon^{\mu\alpha\beta\gamma} p_{1\gamma}$  for  $s_{23} + s_2 + s_3 = \text{odd}$

(There are cases,where P can be skipped.That happens after contraction of P with a pure spin tensor on either side of P,when both are orthogonal to the same momentum.In these cases the pure spin tensor takes over the symmetrization(see the relations above and example 3))

The tensors  $P_{\dots}^{(s_{23})}(p_1) [\dots] \epsilon_{\dots}^{*(s_2)}(p_2, m_2) \epsilon_{\dots}^{*(s_3)}(p_3, m_3)$  may have to be contracted in order to get a tensor of rank  $s_{23}$ ,orthogonal to  $p_1$ .  
The number of contractions is  $1/2(2s_{23} + s_2 + s_3 - s_{23})$  for  $[\dots]=1$ , otherwise  $1/2(2s_{23} + 4 + 1 + s_2 + s_3 - s_{23})$

A contraction also may be necessary for  $[\dots] \tilde{t}_{\dots}^{(l_{23})}(p_1) \chi_{\dots}^{(s_{23})}(p_1)$  to get a tensor of rank  $(s_1)$ ,orthogonal to  $p_1$ .  
The number of contractions here is  $1/2(l_{23} + s_{23} - s_1)$  for  $[\dots]=1$ , otherwise  $1/2(4 + 1 + l_{23} + s_{23} - s_1)$

Example 1:  $3^- \rightarrow 2^+ + 1^-$  with  $s_{23} = 3, l_{23} = 4$

Construction of  $\chi^{(s_{23})}$ :

$s_2 + s_3 + s_{23}=6=\text{even} \rightarrow [\dots]=1$ ; no. of contract.= $1/2(2 \times 3 + 2 + 1 - 3)=3$

$$\chi_{\nu_1 \nu_2 \nu_3}^{(3)}(p_1) = P_{\nu_1 \nu_2 \nu_3 \nu_1' \nu_2' \nu_3'}^{(3)}(p_1) \epsilon_{\nu_1' \nu_2'}^{*(2)}(p_2, m_2) \epsilon_{\nu_3'}^{*(1)}(p_3, m_3)$$

The three contracted indices are highlighted;there is no other way to contract to  $\chi^{(3)}(p_1)$ .

Construction of the LS-amplitude:

$s_{23} + l_{23} + s_1=10=\text{even} \rightarrow [\dots]=1$ ;no. of contr.= $1/2(4+3-3)=2$

$$\rightarrow \tilde{t}_{\mu_1 \mu_2}^{(4)\alpha\beta}(p_1) \chi_{\alpha\beta\mu_3}^{(3)}(p_1)$$

The two contracted indices are highlighted;there is no other way to contract to rank 3;the not contracted indices are renamed.

The total amplitude is:

$$A = \Lambda_{43} \epsilon^{(3)\mu_1 \mu_2 \mu_3}(p_1, m_1) \tilde{t}_{\mu_1 \mu_2}^{(4)\alpha\beta}(p_1) P_{\alpha\beta\mu_3 \nu_1' \nu_2' \nu_3'}^{(3)}(p_1) \epsilon_{\nu_1' \nu_2'}^{*(2)}(p_2, m_2) \epsilon_{\nu_3'}^{*(1)}(p_3, m_3)$$

Example 2:  $3^- \rightarrow 2^+ + 1^-$  with  $s_{23} = 2, l_{23} = 4$

Construction of  $\chi^{(s_{23})}$ :

$s_2 + s_3 + s_{23}=5=\text{odd} \rightarrow$  no. of contr.= $1/2(2 \times 2 + 4 + 1 + 2 + 1 - 2)=5$

$$\chi_{\nu_1 \nu_2}^{(2)}(p_1) = P_{\nu_1 \nu_2 \nu_1' \nu_2'}^{(2)}(p_1) \epsilon_{\nu_1' \alpha_1' \alpha_2' \alpha_3'}^{\nu_1' \alpha_1' \alpha_2' \alpha_3'} p_{1\alpha_1'} \epsilon_{\alpha_2'}^{*(2)\nu_2'}(p_2, m_2) \epsilon_{\alpha_3'}^{*(1)}(p_3, m_3)$$

Firstly,the four indices of  $\epsilon$ ,i.e. $\nu_1' \alpha_1' \alpha_2' \alpha_3'$ ,must be contracted with the four different tensors,because all tensors are symmetric and a double contraction with the same tensor gives zero;the fifth contraction is between  $P^{(2)}$  and

$\epsilon^{*(2)}$ ; there is no other way to contract to rank 2.

Construction of the LS-amplitude:

$$s_{23} + l_{23} + s_1 = 2 + 4 + 3 = 9 = \text{odd} \rightarrow \text{no. of contr.} = 1/2(4 + 1 + 4 + 2 - 3) = 4$$

$$\rightarrow \epsilon^{\mu_1 \alpha_1 \alpha_2 \alpha_3} p_{1\alpha_1} \tilde{t}_{\alpha_2}^{(4)\beta\mu_2\mu_3}(p_1) \chi_{\alpha_3\beta}^{(2)}(p_1)$$

Firstly, three indices of  $\epsilon$ , i.e.  $\alpha_1\alpha_2\alpha_3$ , must be contracted with three different tensors, because all tensors are symmetric and a double contraction gives zero; the fourth contraction is between  $\tilde{t}^{(4)}$  and  $\chi^{(2)}$ ; there is only one possibility to achieve 4 contractions; the not contracted indices have been renamed.

The total amplitude is:

$$A = A_{42} \epsilon_{\mu_1\mu_2\mu_3}^{(3)}(p_1, m_1) \epsilon^{\mu_1\alpha_1\alpha_2\alpha_3} p_{1\alpha_1} \tilde{t}_{\alpha_2}^{(4)\beta\mu_2\mu_3}(p_1) P_{\alpha_3\beta\nu'_1\nu'_2}^{(2)}(p_1) \epsilon^{\nu'_1\alpha'_1\alpha'_2\alpha'_3} p_{1\alpha'_1} \times \\ \times \epsilon_{\alpha'_2}^{*(2)\nu'_2}(p_2, m_2) \epsilon_{\alpha'_3}^{*(1)}(p_3, m_3)$$

### 3 Amplitude for $1 \rightarrow 2 + 3; 2 \rightarrow 4 + 5; 3 \rightarrow 6 + 7$

Notation:

$2(s_2, m_2) \rightarrow 4(s_4, m_4) + 5(s_5, m_5)$ ;  $s_{45}$  = spin of the 2-particle system;  $l_{45}$  = relative angular momentum of the 2-particle system

$3(s_3, m_3) \rightarrow 6(s_6, m_6) + 7(s_7, m_7)$ ;  $s_{67}$  = spin of the 2-particle system;  $l_{67}$  = relative angular momentum of the 2-particle system

Four momenta:  $p_1, p_2, p_3, p_4, p_5, p_6, p_7$ ;  $p_1 = p_2 + p_3$ ;  $p_2 = p_4 + p_5$ ;  $p_3 = p_6 + p_7$

Treat the decay  $2 \rightarrow 4 + 5$  like  $1 \rightarrow 2 + 3$

$$\Rightarrow A \propto \epsilon_{\dots}^{*(s_2)\dots}(p_2, m_2) [\dots] \tilde{t}^{(l_{45})}(p_2) P^{(s_{45})}(p_2) [\dots] \epsilon_{\dots}^{*(s_4)\dots}(p_4, m_4) \epsilon_{\dots}^{*(s_5)\dots}(p_5, m_5)$$

Do the same with  $3 \rightarrow 6 + 7$ :

$$\Rightarrow A \propto \epsilon_{\dots}^{*(s_3)\dots}(p_3, m_3) [\dots] \tilde{t}^{(l_{67})}(p_3) P^{(s_{67})}(p_3) [\dots] \epsilon_{\dots}^{*(s_6)\dots}(p_6, m_6) \epsilon_{\dots}^{*(s_7)\dots}(p_7, m_7)$$

Multiply these amplitudes with the amplitude of  $1 \rightarrow 2 + 3 \Rightarrow$  the terms  $\dots \epsilon^{*(s_2)}(p_2, m_2) \times \epsilon^{(s_2)}(p_2, m_2) \dots$  and  $\dots \epsilon^{*(s_3)}(p_3, m_3) \times \epsilon^{(s_3)}(p_3, m_3) \dots$  appear.

Coherent summation over  $m_2$  and  $m_3$ :

$$\sum_{m_2} \epsilon^{*(s_2)}(p_2, m_2) \times \epsilon^{(s_2)}(p_2, m_2) = P^{(s_2)}(p_2)$$

$$\sum_{m_3} \epsilon^{*(s_3)}(p_3, m_3) \times \epsilon^{(s_3)}(p_3, m_3) = P^{(s_3)}(p_3)$$

Total amplitude:

$$A \propto A_{l_{23}s_{23}l_{45}s_{45}l_{67}s_{67}} \epsilon^{(s_1)}(p_1, m_1) [\dots] \tilde{t}^{(l_{23})}(p_1) P^{(s_{23})}(p_1) \times P^{(s_2)}(p_2) [\dots] \tilde{t}^{(l_{45})}(p_2) P^{(s_{45})}(p_2) [\dots] \epsilon^{*(s_4)}(p_4, m_4) \times \\ \epsilon^{*(s_5)}(p_5, m_5) \times P^{(s_3)}(p_3) [\dots] \tilde{t}^{(l_{67})}(p_3) P^{(s_{67})}(p_3) [\dots] \epsilon^{*(s_6)}(p_6, m_6) \epsilon^{*(s_7)}(p_7, m_7)$$

(Depending on the contributing spins,  $[\dots]$  have to be inserted)

Example 3:  $1^- \rightarrow 1^- + 2^+; 1^- \rightarrow 0^- + 0^-; 2^+ \rightarrow 0^- + 0^-$   
 $s_{23} = 3, l_{23} = 4, s_{45} = 0, l_{45} = 1, s_{67} = 0, l_{67} = 2$

$$\begin{aligned} s_1 + l_{23} + s_{23} &= 8 = \text{even} \rightarrow [\dots] = 1 \\ s_2 + l_{45} + s_{45} &= 2 = \text{even} \rightarrow [\dots] = 1 \\ s_3 + l_{67} + s_{67} &= 4 = \text{even} \rightarrow [\dots] = 1 \\ s_2 + s_3 + s_{23} &= 6 = \text{even} \rightarrow [\dots] = 1 \\ s_4 + s_5 + s_{45} &= 0 = \text{even} \rightarrow [\dots] = 1 \\ s_6 + s_7 + s_{67} &= 0 = \text{even} \rightarrow [\dots] = 1 \end{aligned}$$

$$\begin{aligned} A &\propto \Lambda_{431020} \epsilon^{(1)\mu}(p_1, m_1) \times 1 \times \tilde{t}_\mu^{(4)\alpha_1\alpha_2\alpha_3}(p_1) P_{\alpha_1\alpha_2\alpha_3\alpha'_1\alpha'_2\alpha'_3}^{(3)}(p_1) \times \\ &P^{(1)\alpha'_1\beta}(p_2) \times 1 \times \tilde{t}_\beta^{(1)}(p_2) P^{(0)}(p_2) \times 1 \times 1 \times P^{(2)\alpha'_2\alpha'_3\beta'_1\beta'_2}(p_3) \times 1 \times \tilde{t}_{\beta'_1\beta'_2}^{(2)}(p_3) P^{(0)}(p_3) \times \\ &1 \times 1 \\ &= \Lambda_{431020} \epsilon^{(1)\mu}(p_1, m_1) \times \tilde{t}_\mu^{(4)\alpha_1\alpha_2\alpha_3}(p_1) P_{\alpha_1\alpha_2\alpha_3\alpha'_1\alpha'_2\alpha'_3}^{(3)}(p_1) \tilde{t}^{(1)\alpha'_1}(p_2) \tilde{t}^{(2)\alpha'_2\alpha'_3}(p_3) \\ &= \Lambda_{431020} \epsilon^{(1)\mu}(p_1, m_1) \tilde{t}_{\mu\alpha'_1\alpha'_2\alpha'_3}^{(4)}(p_1) \tilde{t}^{(1)\alpha'_1}(p_2) \tilde{t}^{(2)\alpha'_2\alpha'_3}(p_3) \\ &\text{with} \\ &\tilde{t}_\mu^{(4)\alpha_1\alpha_2\alpha_3}(p_1) P_{\alpha_1\alpha_2\alpha_3\alpha'_1\alpha'_2\alpha'_3}^{(3)}(p_1) = \tilde{t}_{\mu\alpha'_1\alpha'_2\alpha'_3}^{(4)}(p_1) \text{(see above)} \end{aligned}$$

That is a case, where the P-tensor can be skipped.

For an extension to half integral spins see [1, 5, 9, 17].

(Note: In case that for  $1 \rightarrow 2 + 3; 2 \rightarrow 4 + 5$  a broad resonance, e.g. particle 2, decaying to  $4 + 5$ , is involved, a dynamical amplitude has to be inserted. In the simplest case (resonance far from thresholds) it has a BW shape

$$\frac{B_{L_{23}}(Q_{123}) B_{L_{45}}(Q_{245})}{m_2^2 - s_{45} - im_a \Gamma(\sqrt{s_{45}})} \text{ (see [4])}$$

with  $m_2$  = nominal mass of particle 2;  $s_{45} = (p_4 + p_5)^2$ ; B = Blatt-Weisskopf barrier factors;  $Q_{abc}$  = magnitude of  $\vec{p}_b$  or  $\vec{p}_c$  in the rest system of a)

## 4 Acknowledgements

For helpful discussions and explanations I thank Michael Williams (MIT/Boston) and Bingsong Zou (CCAST, Beijing).

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