Differential cross section for $\overline{p}p \to \omega\pi^0$ in tensor formalism,v2

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Abstract

Formulae for the differential cross sections for $\overline{p}p \to \omega\pi^0$ with ω decaying into $\pi^+\pi^-\pi^0$ and into $\pi^0\gamma$ are derived using the tensor formalism(Rarita-Schwinger)

1 Introduction

Increasingly,modern PWA-analyses use the full relativistic invariant tensor formalism(Rarita-Schwinger) [5, 1, 3, 4, 8] to express the amplitudes.The reason is obvious:Standard methods(Helicity-,Zemach-formalisms)disregard the relativistic effects connected with the speed of resonances.In case of low speed,the effect is negligeable,but in many cases essential effects occur[5].In addition,the tensor formalism ensures the correct behaviour of the partial wave amplitudes near thresholds.An introduction into the formalism is given in [1, 3, 5, 6, 7],based on [10, 12, 13, 9, 11].It is easy to express the amplitudes in $\overline{p}p$ reactions in this formalism.As an example are given here the differential cross sections for the reactions $\overline{p}p \to \pi^0\omega, \omega \to \pi^+\pi^-\pi^0$and $\omega \to \pi^0\gamma$.For details of kinematics and so on see [14].The nomenclature corresponds to the one used in the qft++ package [2].

2 Amplitude for $\overline{p}p \to \pi^0\omega, \omega \to \pi^+\pi^-\pi^0$

The particles are numbered as follows: $\overline{p}(1)p(2) \to \pi^0(3)\omega(4); \omega(4) \to \pi^+(5)\pi^-(6)\pi^0(7)$

Relevant 4-momenta are:$p = p_1 + p_2 = p_3 + p_4; p_\omega = p_5 + p_6 + p_7; r_{12} = p_1 - p_2; r_{34} = p_3 - p_4; s = p^2$

J: Total angular momentum of the initial J_M^{PC} state with the projections M (restricted to 0,±1);$m_1, m_2 = \pm 1/2$(spin projections of $\overline{p}, p$);$m_\omega = 0, \pm 1$ (spin projections of the omega)
All projections refer to the same z-axis,e.g. the beam direction of the antiproton.
The total spins are S_{12}(0,1) and S_{34}(1), the orbital momenta L_{12}, L_{34} are described by $\tilde{T}(p), \tilde{t}(p)$.

The differential cross section is given as function of $\cos\Theta, \cos\theta, \phi'$,where Θ is the production angle of ω, θ and ϕ' are the angles of the normal to the 3π decay plane,ϕ' is given relative to the $\pi^0\omega$ decay plane.

The differential cross section is integrated over the Euler-angle γ and over the internal degrees of freedom of the Dalitz-decay,e.g. m_{56} and m_{57}.

$$\frac{d^3\sigma}{d\cos\Theta\, d\cos\theta\, d\phi'} \propto$$
$$\propto \sum_{m_1 m_2} \Big| \sum_J \sum_{m_\omega} A(\overline{p}(m_1)p(m_2) \to J_M^{PC})\, A(J_M^{PC} \to \pi^0\omega_{m_\omega})\, A(\omega_{m_\omega} \to \pi^+\pi^-\pi^0)\, \Big|^2 (M = m_1 - m_2) \tag{1}$$

$$\begin{aligned}
&\propto \Big| \sum_J \sum_{m_\omega} A(\overline{p}(1/2)p(-1/2) \to J_M^{PC})\, A(J_M^{PC} \to \pi^0\omega_{m_\omega})\, A(\omega_{m_\omega} \to \pi^+\pi^-\pi^0)\, \Big|^2 + \\
&\quad (m_1 = 1/2, m_2 = -1/2, M = 0; S_{12} = 0, 1) \\
&\quad + \Big| \sum_J \sum_{m_\omega} A(\overline{p}(-1/2)p(1/2) \to J_M^{PC})\, A(J_M^{PC} \to \pi^0\omega_{m_\omega})\, A(\omega_{m_\omega} \to \pi^+\pi^-\pi^0)\, \Big|^2 + \\
&\quad (m_1 = -1/2, m_2 = 1/2, M = 0; S_{12} = 0, 1) \\
&\quad + \Big| \sum_J \sum_{m_\omega} A(\overline{p}(1/2)p(1/2) \to J_M^{PC})\, A(J_M^{PC} \to \pi^0\omega_{m_\omega})\, A(\omega_{m_\omega} \to \pi^+\pi^-\pi^0)\, \Big|^2 + \\
&\quad (m_1 = 1/2, m_2 = 1/2, M = 1; S_{12} = 1) \\
&\quad + \Big| \sum_J \sum_{m_\omega} A(\overline{p}(-1/2)p(-1/2) \to J_M^{PC})\, A(J_M^{PC} \to \pi^0\omega_{m_\omega})\, A(\omega_{m_\omega} \to \pi^+\pi^-\pi^0)\, \Big|^2 \\
&\quad (m_1 = -1/2, m_2 = -1/2, M = -1; S_{12} = 1)
\end{aligned} \tag{2}$$

For the kinematical factors see[14].

Only J^{PC}-states are allowed which fulfill the conservation of parity($P_{12} = (-1)^{L_{12}+1} = P_{34} = (-1)^{L_{34}}$) and of C-parity($C_{12} = (-1)^{L_{12}+S_{12}} = C_{34} = -1$).As a consequence,for M=0 only J-odd states are permitted.(see also [14],[15]).

The amplitudes are expressed in the tensor formalism ([16, 1, 3]).Examples are:

$A(\overline{p}(1/2)p(1/2) \to 1_1^{--}) \;=\; \epsilon_\mu^{(1)*}(p,1)\,(\Lambda_{0,1}^{1^-}\psi^{(1)\mu}(p_1,1/2,p_2,1/2)\,\tilde{T}^{(0)}(p)+$
$+\Lambda_{2,1}^{1^-}\psi_\nu^{(1)}(p_1,1/2,p_2,1/2)\,\tilde{T}^{(2)\mu\nu}(p))$
$A(1_1^{--} \to \pi^0\omega_{m_\omega}) \;=\; \Lambda_{1,1}^{1^-}\varepsilon^{\mu\beta\gamma\delta}\,p_\delta\,\tilde{t}_\beta^{(1)}\,\epsilon_\gamma^{(1)*}(p_\omega,m_\omega)\,\epsilon_\mu^{(1)}(p,1)$
$A(\omega_{m_\omega} \to \pi^+\pi^-\pi^0) \;=\; N\,\imath\,\varepsilon^{\mu\nu\alpha\beta}\,(p_5)_\nu\,(p_6)_\alpha\,(p_7)_\beta\,\epsilon_\mu^{(1)}(p_\omega,m_\omega)$($\omega$ decay amplitude,see [1])

with
$\epsilon_\mu^{(1)}(p,M=1)$ the spin-1 wave function for the initial state 1_1^{--},
$\epsilon_\mu^{(1)}(p_\omega,m_\omega)$ the spin-wave function of the omega [5],
$\psi_\mu^{(1)}(p_1,m_1,p_2,m_2)$ the spin-1 wave function for the $\overline{p}p$-system [4]
and the coupling constants $\Lambda_{L_{12},S_{12}}^{J^P}, \Lambda_{L_{34},S_{34}}^{J^P}$.

M, m_1, m_2 are the spin projections and refer to a fixed x,y,z-coordinate-system.The z-axis can have any direction,but is chosen here as the direction of the antiproton beam.
Introduction of the tensor wave functions yields:

$$
\begin{aligned}
&\frac{d^3\sigma}{d\cos\Theta\, d\cos\theta\, d\phi'} \propto \\
&\propto 2\Big|\sum_{J_{odd}} \big(\Lambda^{J^+}_{J,0,J-1,1}\,\tilde{t}^{(J-1)}(p)\,\omega^{\mu_1}(p_\omega) \;+\; \Lambda^{J^+}_{J,0,J+1,1}\,\tilde{t}^{(J+1)}(p)\,\omega^{\mu_1}(p_\omega)\big)\,P^{(J)}_{\mu_1..\nu_1..}(p)\psi^{(0)}(1/2,-1/2)\,\tilde{T}^{(J+1)}(p)\,\Big|^2 + \\
&(M=0, S_{12}=0(singlett)) \\
&+2\Big|\sum_{J_{odd}} \varepsilon^{\mu_1\beta\gamma\delta}p_\delta\,\tilde{t}^{(J)}_\beta(p)\,\omega_\gamma(p_\omega)\,P^{(J)}_{\mu_1..\nu_1..}(p)\big(\,\Lambda^{J^-}_{J-1,1,J,1}\,\psi^{(1)}(-1/2,1/2)\tilde{T}^{(J-1)}(p) + \Lambda^{J^-}_{J+1,1,J,1}\psi^{(1)}(1/2,-1/2)\tilde{T}^{(J+1)}(p)\big)\,\Big|^2 + \\
&(M=0, S_{12}=1(triplett)) \\
&+\Big|\sum_{J_{odd}} \varepsilon^{\mu_1\beta\gamma\delta}p_\delta\,\tilde{t}^{(J)}_\beta(p)\,\omega_\gamma(p_\omega)P^{(J)}_{\mu_1..\nu_1..}(p)\big(\Lambda^{J^-}_{J-1,1,J,1}\,\psi^{(1)}(1/2,1/2)\,\tilde{T}^{(J-1)}(p) + \Lambda^{J^-}_{J+1,1,J,1}\psi^{(1)}(1/2,1/2)\,\tilde{T}^{(J+1)}(p)\big) + \\
&+\sum_{J_{even}} \big(\Lambda^{J^-}_{J,1,J-1,1}\,\tilde{t}^{(J-1)}(p)\,\omega^{\mu_1}(p_\omega) + \Lambda^{J^-}_{J,1,J+1,1}\,\tilde{t}^{(J+1)}(p)\,\omega^{\mu_1}(p_\omega)\big)\,P^{(J)}_{\mu_1..\nu_1..}(p)\,\varepsilon^{\nu_1\beta\gamma\delta}\,p_\delta\,\psi^{(1)}_\beta(1/2,1/2)\,\tilde{T}^{(J)}_\gamma(p)\,\Big|^2 + \\
&(M=1, S_{12}=1(triplett)) \\
&+\Big|\sum_{J_{odd}} \varepsilon^{\mu_1\beta\gamma\delta}p_\delta\,\tilde{t}^{(J)}_\beta(p)\,\omega_\gamma(p_\omega)P^{(J)}_{\mu_1..\nu_1..}(p)\big(\Lambda^{J^-}_{J-1,1,J,1}\,\psi^{(1)}(-1/2,-1/2)\,\tilde{T}^{(J-1)}(p) + \Lambda^{J^-}_{J+1,1,J,1}\psi^{(1)}(-1/2,-1/2)\,\tilde{T}^{(J+1)}(p)\big) + \\
&+\sum_{J_{even}} \big(\Lambda^{J^-}_{J,1,J-1,1}\,\tilde{t}^{(J-1)}(p)\,\omega^{\mu_1}(p_\omega) + \Lambda^{J^-}_{J,1,J+1,1}\,\tilde{t}^{(J+1)}(p)\,\omega^{\mu_1}(p_\omega)\big)\,P^{(J)}_{\mu_1..\nu_1..}(p)\,\varepsilon^{\nu_1\beta\gamma\delta}\,p_\delta\,\psi^{(1)}_\beta(-1/2,-1/2)\,\tilde{T}^{(J)}_\gamma(p)\,\Big|^2 \\
&(M=-1, S_{12}=1(triplett))
\end{aligned}
\tag{3}
$$

(Attention:Not all indices are explicitly given,please check the forthcoming examples;the arguments p_1, p_2 in the $\overline{p}p$ wave functions were skipped)

with:
$\tilde{T}^{(0)}(p) = 1$
$\tilde{T}^{(1)}_{\mu}(p) \,=\, \tilde{g}_{\mu\nu}(p)\, r^{\nu}_{12} \,=\, (\tilde{r}_{12})_{\mu}$
$\tilde{T}^{(2)}_{\mu\nu}(p) \,=\, [(\tilde{r}_{12})_{\mu}\,(\tilde{r}_{12})_{\nu} \,-\, 1/3(\tilde{r}_{12}\cdot\tilde{r}_{12}))\,\tilde{g}_{\mu\nu}(p)]$

$\tilde{t}^{(0)}(p) = 1$
$\tilde{t}^{(1)}_{\mu}(p) \,=\, \tilde{g}_{\mu\nu}(p)\, r^{\nu}_{34} \,=\, (\tilde{r}_{34})_{\mu}$
$\tilde{t}^{(2)}_{\mu\nu}(p) \,=\, [(\tilde{r}_{34})_{\mu}\,(\tilde{r}_{34})_{\nu} \,-\, 1/3(\tilde{r}_{34}\cdot\tilde{r}_{34})\,\tilde{g}_{\mu\nu}(p)]$
and so on.(see[1],[3])

with $\tilde{g}_{\mu\nu}(p) \,=\, g_{\mu\nu} \,-\, \frac{p_\mu p_\nu}{p^2}$; $g_{\mu\nu} = g^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$

$P^{(1)}_{\mu_1\nu_1}(p) \,=\, \sum_M \epsilon^{(1)}_{\mu_1}(p,M)\,\epsilon^{(1)*}_{\nu_1}(p,M) \,=\, -\tilde{g}_{\mu_1\nu_1}(p)$
$P^{(2)}_{\mu_1\mu_2\nu_1\nu_2}(p) \,=\, \sum_M \epsilon^{(2)}_{\mu_1\mu_2}(p,M)\,\epsilon^{(2)*}_{\nu_1\nu_2}(p,M) \,=\, 1/2(\tilde{g}_{\mu_1\nu_1}\,\tilde{g}_{\mu_2\nu_2} + \tilde{g}_{\mu_1\nu_2}\,\tilde{g}_{\mu_2\nu_1}\,) - 1/3\tilde{g}_{\mu_1\mu_2}\,\tilde{g}_{\nu_1\nu_2}$
and so on(see [1, 3]

$\psi^{(0)}(p_1,m_1,p_2,m_2) \,=\, \bar{v}(p_1,m_1)\,\gamma_5\,u(p_2,m_2)$ (spin wave function for $\overline{p}p$-system with $S_{12} = 0$)
$\psi^{(1)\mu}(p_1,m_1,p_2,m_2) \,=\, \bar{v}(p_1,m_1)\,(\gamma^\mu + \frac{r^{\mu}_{12}}{\sqrt{s}+m_{\overline{p}}+m_p})\,u(p_2,m_2)$ (spin wave function for $\overline{p}p$-system with $S_{12} = 1$) (equivalent to [4],but with v and u interchanged;here $\overline{p}$ and p are incoming particles)
The Dirac spinor $\bar{v}(p_1,m_1)$ describes an incoming $\overline{p}$ with four momentum p_1,and u(p_2,m_2) the incoming p with four momentum p_2 with

$\bar{v} = v\gamma^0$ and e.g.
u(p,m=1/2) $= N_1 \begin{pmatrix} 1 \\ 0 \\ \frac{c(p_z)}{(E+mc^2)} \\ \frac{c(p_x+\imath p_y)}{(E+mc^2)} \end{pmatrix}$ v(p,m=1/2) $= N_1 \begin{pmatrix} \frac{c(p_x-\imath p_y)}{E+mc^2} \\ \frac{c(-p_z)}{E+mc^2} \\ 0 \\ 1 \end{pmatrix}$
$N_1 = \sqrt{(|E| + mc^2)/c}$
This form assumes,that the quantization axis is the z-axis.Note that u/v are not eigen states of Σ_z(see [17])

$\omega_\mu(p_\omega) \,=\, N\,P^{(1)}_{\mu\nu}(p_\omega)\,\varepsilon^{\nu\alpha\beta\gamma}\,(p_5)_\alpha\,(p_6)_\beta\,(p_7)_\gamma$ (see [1, 3])

$\Lambda_{L_{12},S_{12},L_{34},S_{34}}$: Coupling constants for production and decay,normalization factor N absorbed

To separate the $S_{12} = 0$ and $S_{12} = 1$ terms with M=0 the relation $|A+B|^2 + |-A+B|^2 = 2|A|^2+2|B|^2$ was used with $\psi^{(0)}(1/2,-1/2) = \psi^{(0)}(-1/2,1/2)$ and $\psi^{(1)}_\mu(1/2,-1/2) = -\psi^{(1)}_\mu(-1/2,1/2)$.

2.1 Examples for amplitudes

Now all indices are explicitly given.

$$\begin{aligned} A_1^{1^-}(M=1) \;=\; & \sum_\omega A(\overline{p}(1/2)p(1/2)\to 1_1^{--})A(1^{--}\to\pi^0\omega_{m_\omega})A(\omega_{m_\omega}\to\pi^+\pi^-\pi^0) = \\ =\; & \varepsilon^{\mu_1\beta\gamma\delta}\,p_\delta\,\tilde{t}_\beta^{(1)}(p)\,\omega_\gamma(p_\omega)\,P^{(1)}_{\mu_1\nu_1}(p)\,\big(\Lambda^{1^-}_{0,1,1,1}\,\psi^{(1)\nu_1}(p_1,1/2,p_2,1/2)\,\tilde{T}^{(0)}(p) + \\ +\; & \Lambda^{1^-}_{2,1,1,1}\,\psi^{(1)}_\alpha(p_1,1/2,p_2,1/2)\,\tilde{T}^{(2)\alpha\nu_1}(p)\big) \end{aligned} \tag{4}$$

$$\begin{aligned} A_1^{3^-}(M=1) \;=\; & \varepsilon^{\mu_1\beta\gamma\delta}\,p_\delta\,\tilde{t}_\beta^{(3)\mu_2\mu_3}(p)\,\omega_\gamma(p_\omega)\,P^{(3)}_{\mu_1\mu_2\mu_3\nu_1\nu_2\nu_3}(p)\big(\Lambda^{3^-}_{2,1,3,1}\,\psi^{(1)\nu_1}(p_1,1/2,p_2,1/2)\,\tilde{T}^{(2)\nu_2\nu_3}(p) \\ +\; & \Lambda^{3^-}_{4,1,3,1}\,\psi^{(1)}_\alpha(p_1,1/2,p_2,1/2)\,\tilde{T}^{(4)\alpha\nu_1\nu_2\nu_3}(p)\big) \end{aligned} \tag{5}$$

$$\begin{aligned} A_0^{1^+}(M=0) \;=\; & \big(\Lambda^{1+}_{1,0,0,1}\,\tilde{t}^{(0)}(p)\,\omega^{\mu_1}(p_\omega) \;+\; \Lambda^{1+}_{1,0,2,1}\,\tilde{t}^{(2)\alpha\mu_1}(p)\,\omega_\alpha(p_\omega)\,\big)\times \\ & \times P^{(1)}_{\mu_1\nu_1}(p)\,\psi^{(0)}(p_1,1/2,p_2,-1/2)\,\tilde{T}^{(1)\nu_1}(p) \end{aligned} \tag{6}$$

$$\begin{aligned} A_0^{3^+}(M=0) \;=\; & \big(\Lambda^{3+}_{3,0,2,1}\,\tilde{t}^{(2)\mu_1\mu_2}(p)\,\omega^{\mu_3}(p_\omega) \;+\; \Lambda^{3+}_{3,0,4,1}\,\tilde{t}^{(4)\alpha\mu_1\mu_2\mu_3}(p)\,\omega_\alpha(p_\omega)\,\big)\times \\ & \times P^{(3)}_{\mu_1\mu_2\mu_3\nu_1\nu_2\nu_3}(p)\,\psi^{(0)}(p_1,1/2,p_2,-1/2)\,\tilde{T}^{(3)\nu_1\nu_2\nu_3}(p) \end{aligned} \tag{7}$$

$$\begin{aligned} A_0^{1^-}(M=0) \;=\; & \varepsilon^{\mu_1\beta\gamma\delta}\,p_\delta\,\tilde{t}_\beta^{(1)}(p)\,\omega_\gamma(p_\omega)\,P^{(1)}_{\mu_1\nu_1}(p)\,\big(\Lambda^{1^-}_{0,1,1,1}\,\psi^{(1)\nu_1}(p_1,1/2,p_2,-1/2)\,\tilde{T}^{(0)}(p) + \\ & + \Lambda^{1^-}_{2,1,1,1}\,\psi^{(1)}_\alpha(p_1,1/2,p_2,-1/2)\,\tilde{T}^{(2)\alpha\nu_1}(p)\big) \end{aligned} \tag{8}$$

$$\begin{aligned} A_1^{2^-}(M=1) \;=\; & \big(\Lambda^{2^-}_{2,1,1,1}\,\tilde{t}^{(1)\mu_1}(p)\,\omega^{\mu_2}(p_\omega) \;+\; \Lambda^{2^-}_{2,1,3,1}\,\tilde{t}^{(3)\alpha\mu_1\mu_2}(p)\,\omega_\alpha(p_\omega)\big)\times \\ & \times P^{(2)}_{\mu_1\mu_2\nu_1\nu_2}(p)\varepsilon^{\nu_1\beta\gamma\delta}p_\delta\psi^{(1)}_\beta(p_1,1/2,p_2,1/2)\,\tilde{T}^{(2)\nu_2}_\gamma(p) \end{aligned} \tag{9}$$

with

$\psi^{(1)\nu} \;=\; g^{\nu\alpha}\,\psi^{(1)}_\alpha$ and

$\tilde{t}_\beta^{(3)\mu_2\mu_3} \;=\; g^{\mu_2\gamma}\,g^{\mu_3\delta}\,\tilde{t}^{(3)}_{\beta\gamma\delta};\quad \tilde{T}_\gamma^{(2)\nu_2} \;=\; g^{\alpha\nu_2}\,\tilde{T}^{(2)}_{\alpha\gamma}$

3 Amplitude for $\overline{p}p \to \pi^0\omega, \omega \to \pi^0\gamma$

The particles are numbered as follows: $\overline{p}(1)p(2) \to \pi^0(3)\omega(4); \omega(4) \to \pi^0(5)\gamma(6)$

Relevant 4-momenta are:$p = p_1+p_2 = p_3+p_4; p_\omega = p_5+p_6; r_{12} = p_1-p_2; r_{34} = p_3 - p_4; r_{56} = p_5 - p_6; s = p^2$

J:Total angular momentum of the initial state with the projections M=0,±1; $m_1, m_2 = \pm 1/2$(spin projections of $\overline{p}, p$);$m_\omega = 0, \pm 1$ (spin projections of the omega);$m_\gamma = \pm 1$(spin projections of the gamma)

The orbital momenta L_{12}, L_{34}, L_{56} are described by $\tilde{T}(p), \tilde{t}(p), \tilde{t}(p_\omega)$

The differential cross section is given as function of $\cos\Theta, \cos\theta, \phi'$,where Θ is the production angle of ω, θ and $\phi'$are the decay angles of the gamma in the ω rest frame,ϕ' is defined relative to the $\pi^0\omega$ decay plane.

$$\frac{d^3\sigma}{d\cos\Theta\, d\cos\theta\, d\phi'} \propto$$
$$\propto \sum_{m_1 m_2} \sum_{m_\gamma=\pm 1} \Big| \sum_J \sum_{m_\omega} A(\overline{p}(m_1)p(m_2) \to J^{PC}_M) A(J^{PC}_M \to \pi^0 \omega_{m_\omega}) A(\omega_{m_\omega} \to \pi^0 \gamma_{m_\gamma}) \Big|^2$$
$$(M = m_1 - m_2) \tag{10}$$

$\propto\ 2\,\big|$...as eq.(3), $\omega_\mu(p_\omega)$ is replaced by $\tilde{\omega}^\mu(p_\omega, m_\gamma = +1)$...$\big|^2 +$
$+\,2\,\big|$...as eq.(3), $\omega_\mu(p_\omega)$is replaced by $\tilde{\omega}^\mu(p_\omega, m_\gamma = +1)$...$\big|^2 +$
$+\big|$...as eq.(3), $\omega_\mu(p_\omega)$is replaced by $\tilde{\omega}^\mu(p_\omega, m_\gamma = +1)$...$\big|^2 +$
$+\big|$...as eq.(3), $\omega_\mu(p_\omega)$is replaced by $\tilde{\omega}^\mu(p_\omega, m_\gamma = +1)$...$\big|^2 +$

$+2\,\big|$...as eq.(3), $\omega_\mu(p_\omega)$is replaced by $\tilde{\omega}^\mu(p_\omega, m_\gamma = -1)$...$\big|^2 +$
$+\,2\,\big|$...as eq.(3), $\omega_\mu(p_\omega)$is replaced by $\tilde{\omega}^\mu(p_\omega, m_\gamma = -1)$...$\big|^2 +$
$+\big|$...as eq.(3), $\omega_\mu(p_\omega)$is replaced by $\tilde{\omega}^\mu(p_\omega, m_\gamma = -1)$...$\big|^2 +$
$+\big|$...as eq.(3), $\omega_\mu(p_\omega)$is replaced by $\tilde{\omega}^\mu(p_\omega, m_\gamma = -1)$...$\big|^2 +$
with
$\tilde{\omega}^\mu(p_\omega, m_\gamma = \pm 1) = \Lambda^{1^-}_\omega\, P^{(1)\mu\nu}(p_\omega)\, \varepsilon_{\nu\alpha\beta\gamma}\, p^\alpha_\omega\, \tilde{t}^{(1)\beta}(p_\omega)\, \epsilon^{(1)\gamma}(p_\gamma, m_\gamma = \pm 1)$
($\Lambda^{1^-}_\omega$ will be absorbed in $\Lambda_{L_{12},S_{12},L_{34},L_{34}}$)

and the spin wave function of the real photon:

$$\epsilon^{(1)\gamma}(p_\gamma, m_\gamma = \pm 1) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \mp\cos\theta\cos\phi' + \imath\sin\phi' \\ \mp\cos\theta\sin\phi' - \imath\cos\phi' \\ \pm\sin\theta) \end{pmatrix}$$

(see [18],page 71)

(Note: In case that for $1 \to 2+3; 2 \to 4+5$ a broad resonance,e.g.particle 2,decaying to $4+5$,is involved,a dynamical amplitude has to be inserted.In the simplest case(resonance far from thresholds) it has a BW shape $\frac{B_{L_{23}}(Q_{123})\,B_{L_{45}}(Q_{245})}{m_2^2 - s_{45} - \imath m_a \Gamma(\sqrt{s_{45}}}$ (see[3]))
with m_2=nominal mass of particle 2;$s_{45} = (p_4 + p_5)^2$;B=Blatt-Weisskopf barrier factors;Q_{abc}=magnitude of $\vec{p_b}$ or $\vec{p_c}$ in the rest system of a)

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