Construction of tensor amplitudes for radiative J/ψ decays

Helmut Koch

February 16, 2015

In the following several methods to construct tensor amplitudes for radiative decays are discussed.[1, 2, 6, 8, 9, 11, 12, 13, 14, 16, 20, 21, 22, 23, 25, 26].
The terminology is as follows:
$1(M_1, p_1, J_1^{P_1}, m_1(\lambda_1), \psi_1) \rightarrow 2(M_2, p_2, J_2^{P_2}, m_2(\lambda_2), \psi_2) + 3(M_3, p_3, J_3^{P_3}, m_3(\lambda_3), \psi_3)$.
M is the rest mass, J^P the spin$^{\text{parity}}$ and m(λ) the spin projection(helicity) of the particle in the canonical(helicity) formalism.The four-momenta are p_1, p_2, p_3.The spin wave functions are ψ_1, ψ_2, ψ_3 with $(p_1\psi_1) = (p_2\psi_2) = (p_3\psi_3) = 0$.
Specifically the reaction $J/\psi(1^-) \rightarrow \gamma(1^-) + X(J^P)$ is treated below.
The four-momenta are $p_1 = p, p_2 = q, p_3 = k$;the masses are $M_1 = M, M_2 = 0, M_3 = M_X$;the spin wave functions are $\psi_1 = \psi$,$\psi_2 = e^+$ and $\psi_3 = \psi_X$.
In order to construct the radiative amplitudes it is often helpful to start with the amplitudes for particle 2 being a massive vector boson.In this case the four-momenta are $p_1 = p, p_2 = q, p_3 = k$ as before;the masses are $M_1 = M, M_2 \neq 0, M_3 = M_X$;the spin wave functions are $\psi_1 = \psi$,$\psi_2 = e$ and $\psi_3 = \psi_X$.
Useful relations concerning the spin wave functions of the particles are summarized in App.A.
It is assumed that in the massive case $M_2 \ll M_1, M_3$,so that M_2 does not appear in the kinematical expressions.Useful kinematical relations concerning the radiative case are given in App.B
In the expressions given below,the particle 3 is treated as a stable particle.In general,it will be a finite width resonance,so that a relativistic Breit-Wigner has to be added.
The tensor amplitudes exhibit automatically the correct threshold behaviour, being proportional to q^L.As a study of the shape of resonances shows,the term is not adequate to describe the measured resonance shape.An additional form factor has to be defined,which dampes the q^L-behaviour(App.C).In

the examples discussed below always truncated Blatt-Weisskopf factors are given,although other form factors(see App.C) might be more adequate for radiative decays.The form factors are given as part of the orbital momentum tensor(see[2]).

In Appendix D the relations between wave functions defined in a cartesian and a sherical coordinate system are discussed,which is of importance in the case,that particle 1 as a vector meson,e.g. J/ψ,is produced in e^+e^- collisions. [2]

Four different methods to write down radiative tensor amplitudes are discussed in the following.

Method 1(Koch) [1]:
Step1:According to [1] the relevant LS-amplitudes are constructed for the case of massive particles($M_2 \neq 0$) with pure tensors.If there are more LS-amplitudes than helicity amplitudes,there is a relation between the LS-amplitudes, so that the total number of independent amplitudes is equal to the number of helicity amplitudes.That is to get a guidance,which amplitudes may be relevant for the radiative case.
Step2:The transition to the case of particle 2 being a real photon is performed by applying the photon gauge:The spin wave function of the massive vector boson(e) is replaced by
$e^+_\mu = e_\mu - \frac{(ke)}{(kq)} q_\mu$ (see[6] and App.A) (1)
e^+_μ is the photon spin wave function,resulting in the projection operator
$P^{(1)}_{\mu\nu}(q) = \sum_{\lambda=\pm 1} e^+_\mu(\lambda) e^{+*}_\nu(\lambda) = -g_{\mu\nu} + \frac{k_\mu q_\nu + k_\nu q_\mu}{(kq)} - \frac{k^2 q_\mu q_\nu}{(kq)^2}$ ([21],p.158) (2)
In addition to $(e^+q) = 0$,the relations $(e^+p) = 0$ and $(e^+k) = 0$ hold.
The resulting amplitudes are again related,so that the total number of amplitudes is equal to the number of amplitudes in the helicity formalism.

Method 2(Chung)[6]:
Step1:Simplified amplitudes are written down for the case of massive particles for every LS-combination.Simplified means that,e.g.,not the pure tensor for the orbital momentum $\tilde{t}^{(2)}_{\mu\nu}$ is used but instead $q_\mu q_\nu$.
For intrinsic spins and parities with $SP = (-1)^{\text{sum of intrinsic spins}} \times P_1 \times P_2 \times P_3 = -1$ the total antisymmetric tensor ϵ appears in the amplitude as in method 1.All possible combinations between the relevant tensors are taken into account.It may well be,that more combinations than in the helicity or LS-formalism appear(see case 3).This does not happen,if the pure tensors are used.

Step2:see Method 1

The resulting amplitudes are in general linear combinations of the amplitudes constructed by method 1.

Method 3(Zou/Bugg)[2]:
All non-zero combinations of spin wave functions,four-momenta,$g_{\mu\nu}$ and the total antisymmetric tensor ϵ are taken into account for all possible orbital angular momenta.Here,the photon wave functions(e^+) are used immediately,which is obvious from the projection operator($g_{\mu\nu} \to g_{\mu\nu}^{(\perp\perp)}$).([2];[11],Ch.14.6; [12],Ch.5.6; [13],Ch.15.6;[16],Ch.6.13;[20];[21],Ch. 6/7).
Usually,the number of amplitudes is greater than the number of independent helicity amplitudes.From the possible amplitudes,accidentally,amplitudes are selected so that their number corresponds to the number of helicity amplitudes.

Method 4(Williams)[19]:
Here the amplitudes are expanded in terms of multipoles.A multipole is formed by adding the photon spin(1^-) and the orbital angular momentum,resulting in a state j^p.j^p can have the values $1^-, 2^+, 3^-$,..(electric multipoles Ej) or the values $j^p = 1^+, 2^-, 3^+$,...(magnetic multipoles Mj).j^p couples then with the spin of the particle $X(J^P)$,in order to produce the mother particle with $\text{spin}^{\text{parity}} = 1^-$.The advantage of this procedure is,that the minimum number of amplitudes(=number of helicity amplitudes) is produced.Again,one starts from the photon wave functions (e^+).

In the following the four methods are discussed for X-states with several J^P values.

Case1:$1^- \to 1^-(\gamma) + 0^+$

Method 1:
Here,$\psi_X = 1$.Because of parity conservation, only L=0 and L=2 contribute.$S_{Tot} = 1, SP = +1$.The helicity method delivers two amplitudes for $M_2 \neq 0$(massive case) and one amplitude for the radiative process($M_2 = M_\gamma = 0$).
Step1:The LS-amplitudes for the massive case are
$A_1 = \psi^* \cdot \tilde{t}^{(0)} \cdot e = \psi^* \times P^{(1)} \times \tilde{t}^{(0)} \times P^{(1)} e \times 1 = -(e\psi^*) \times B_0$
$A_2 = \psi^* \cdot \tilde{t}^{(2)} \cdot e = \psi^* \times P^{(1)} \times \tilde{t}^{(2)} \times P^{(1)} e \times 1 =$
$= (-2(1 + \frac{M_X^2}{M^2})(q\psi^*)(ke) - \frac{1}{3}\frac{(M^2-M_X^2)^2}{M^2}(e\psi^*)) \times B_2$.
The dot means:The tensors left and right of the dot are contracted,with

(ab)=$a \cdot b$.
B_0 and B_2 are truncated Blatt-Weisskopf factors(the q^L dependence is eliminated,as it is contained in the tensor amplitude).
(see Appendix C and [8],Ch.2;[9],Ch.4.9/App. A1;[18],App. A4;[17],Ch.2.3).

Step2:With the replacement $e \rightarrow e^+$ for the radiative case and with $M_2 = 0$ one obtains
$A_1^{`} = -(e^+\psi^*) \times B_0$
$A_2^{`} = \frac{1}{3}\frac{(M^2-M_X^2)^2}{M^2} \times B_2/B_0 \times A_1^{`}$
using $(ke^+) = 0$
In the radiative case only one independent amplitude remains,if the Blatt-Weisskopf factors are nearly equal.

Method 2:This case is not discussed in [6].

Method 3:
Here only the radiative process is considered.The tensors,from which the amplitudes have to be built,are $g_{\mu\nu}, \psi, e^+, \psi_X = 1$(L=0) and $g_{\mu\nu}, q, q, \psi, e^+, \psi_X = 1$(L=2).$\epsilon_{\alpha\beta\gamma\delta}p^\alpha$ does not contribute,as SP $= +1$.
Two amplitudes are possible:
$A_1^{'} = (e^+\psi^*) \times B_0$
$A_2^{'} = \frac{(M^2-M_X^2)^2}{3M^2} A_1^{'} \times B_2/B_0$
For $B_2 \approx B_0$,only $A_1^{'}$ is independent.Otherwise,both amplitudes have to be used.

Method 4:
Only $j^p = 1^-$(E1) couples with 0^+ to J/ψ.There is only one amplitude:
$A^{'} \propto \psi^*_\mu P^{(1)\mu\alpha} \times 1 \times P^{(1)}_{\alpha\alpha'} t^{(0)} \times e^{+\alpha'} = (e^+\psi^*) \times B_0$

Resume:All four methods yield the same amplitudes.

Case 2:$1^- \rightarrow 1^-(\gamma) + 0^-$

Here again ψ_X=1.$S_{Tot} = 1, L = 1, SP = -1$.The helicity method delivers one amplitude for $M_2 \neq 0$(massive case) and one amplitude for the radiative process.

Method 1:
Step1: The LS-amplitude for the massive case is
$A_1 = (p\tilde{t}^{(1)}e\psi^*) \equiv \psi^* P^{(1)} \epsilon p \tilde{t}^{(1)} P^{(1)} e \times 1 = -2(pqe\psi^*) \times B_1$

with (abcd)=$\epsilon_{\alpha\beta\gamma\delta}a^{\alpha}b^{\beta}c^{\gamma}d^{\delta}$.

Step2:With the replacement $e \to e^{+}$ for the radiative case and with $M_2 = 0$ one obtains
$A_1^{'} = -2(pqe^{+}\psi^{*}) \times B_1$

Method 2:This case is not discussed in [6]

Method 3:
The tensors,from which the amplitude has to be built,are $q, \psi, e^{+}, \psi_X = 1$ and $\epsilon_{\alpha\beta\gamma\delta}p^{\alpha}(SP = -1)$.Only one possibility remains:
$A_1^{'} = (pqe^{+}\psi^{*}) \times B_1$

Method 4:
The multipole $j^p = 1^{+}$ (M1) couples with 0^{-} to 1^{-}.The only amplitude is
$A_1^{'} = \psi^{\mu *} \times P^{(1)}_{\mu\alpha'} \times 1 \times P^{(1)\alpha'\alpha}\epsilon_{\alpha\beta\gamma\delta}p^{\delta}t^{(1)\beta}e^{+\gamma} = 2(pqe^{+}\psi^{*}) \times B_1$

Resume:All methods yield the same amplitudes.

Case 3:$1^{-} \to 1^{-}(\gamma) + 1^{+}$

For the massive case the combinations $(L^P = 2^{+}; S_{Tot} = 2^{-})(A_1)$,$(L^P = 2^{+}; S_{Tot} = 1^{-})(A_2)$ and $(L^P = 0^{+}; S_{Tot} = 1^{-})(A_3)$ contribute.SL=-1.
Both the LS-formalism as well as the helicity formalism deliver three amplitudes for $M_2 \neq 0$(massive case).The helicity formalism delivers two amplitudes for the radiative process.

Method 1:
Step1:The LS-amplitudes for the massive case are
$A_1 = (\psi^{*} \cdot \epsilon p\tilde{t}^{(2)} \cdot e\psi_X) =$
$= [-(1 + \frac{M_X^2}{M^2})(q\psi_x)(pqe\psi^{*}) - ((1 - \frac{M_X^2}{M^2})(ke)(pq\psi_X\psi^{*})] \times B_2$
$A_2 = (\psi^{*} \cdot \tilde{t}^{(2)} \cdot \epsilon pe\psi_X) = [-4(q\psi^{*})(pqe\psi_X) - 4/3(\vec{q}\vec{q})(pe\psi_X\psi^{*})] \times B_2$
$A_3 = (\psi^{*} \cdot \epsilon pe\psi_X) = -(pe\psi_X\psi^{*}) \times B_0$

Step2:The replacement $e \to e^{+}$ and $M_2 = 0$ yields the amplitudes for the radiative process:
$A_1^{'} = [-(1 + \frac{M_X^2}{M^2})(q\psi_x)(pqe^{+}\psi^{*})] \times B_2$
$A_2^{'} = [-4(q\psi^{*})(pqe^{+}\psi_X) - 4/3(\vec{q}\vec{q})(pe^{+}\psi_X\psi^{*})] \times B_2$
$A_3^{'} = -(pe^{+}\psi_X\psi^{*}) \times B_0$

There exists a relation between the amplitudes:
$(M^2 - M_X^2)A_1^{'}/B_2 + 8A_2^{'}/B_2 - \frac{4}{3}\frac{(M^2-M_X^2)^2}{M^2}A_3^{'}/B_0 = 0$
so that only two independent amplitudes remain,in case that the Blatt-Weisskopf factors are nearly equal.Otherwise,all three amplitudes have to be used.
For the relation between the amplitudes the method discussed in [6](App.A) is used.The relation is derived in the CM-system.As the coefficients of the amplitudes are Lorentz-invariant,the relation is valid in all coordinate systems.

Method 2:
Step1:Here truncated tensors are used.The spin one tensor for the $\gamma\psi_X$-system is replaced by $\rho^-_{\alpha\beta} = \psi_{X\alpha}e_\beta - e_\alpha\psi_{X\beta}$([6],equ.(30)),the spin two tensor for the $\gamma\psi_X$-system is replaced by $\rho^+_{\alpha\beta} = \psi_{X\alpha}e_\beta + e_\alpha\psi_{X\beta}$ and the L=2 orbital tensor([6],equ.(35)) is replaced by $t_{\alpha\beta} = r_\alpha r_\beta$ with r = q - k.
The relevant tensors for constructing the amplitudes are ψ, ρ^-(contains e and ψ_X), $t, \epsilon p$ (L=2,S=1),ψ, ρ^+(contains e and ψ_X), $t, \epsilon p$ (L=2,S=2) and ψ, ρ^-(contains e and ψ_X), ϵp(L=0,S=1).
In contrast to the LS-and helicity formalism (see Method 1) <u>four</u> possible amplitudes appear.
$A_1 = (p\rho^-\psi^*) = (pe\psi_X\psi^*) \times B_0$ (L=0,S=1)
$A_2 = 1/2(p\rho^- t \cdot \psi^*) = (r\psi^*)(pre\psi_X) \times B_2$ (L=2,S=1)
$\tilde{A}_2 = 1/2(p\rho^- \cdot t\psi^*) = (r\psi_X)(pre\psi^*) - (re)(pr\psi_X\psi^*) \times B_2$ (L=2,S=1)
$A_3 = 1/2(p\rho^+ \cdot t\psi^*) = (r\psi_X)(pre\psi^*) + (re)(pr\psi_X\psi^*) \times B_2$ (L=2,S=2)

The amplitudes are not independent: $M\tilde{A}_2/B_2 = Mr^2A_3/B_2 + MA_2/B_2 + (q_0 - k_0)A_1/B_0$(see [6]App.A)
This,however,is only true,if the Blatt-Weisskopf factors are nearly equal.

Step2:The replacement $e \rightarrow e^+$ yields three amplitudes for the photon case
$A_1^{'} = (pe^+\psi_X\psi^*) \times B_0$ (L=0,S=1)
$A_2^{'} = (r\psi^*)(pqe^+\psi_X) \times B_2 = 4(q\psi^*)(pqe^+\psi_X) \times B_2$ (L=2,S=1)
$A_3^{'} = (r\psi_X)(pre^+\psi^*) = 2(q\psi_X)(pqe^+\psi^*) \times B_2$ (L=2,S=2)
A_3 and $\tilde{A}_2$ reduce to $A_3^{'}$.
Again,they are not independent: $(M^2 - m_X^2)^2A_1^{'}/B_0 + M^2A_2^{'}/B_2 + (M^2 + m_X^2)A_3^{'}/B_2 = 0$ (see[6],App.B),so that two independent amplitudes remain.
This,however,is only true,if the Blatt-Weisskopf factors are nearly equal for all amplitudes.Otherwise,all three amplitudes have to be used.

Method 3:

The tensors,from which the amplitudes have to be built are $e^+, \psi, \psi_X, \epsilon p$(L=0) and $q, q, e^+, \psi, \psi_X, \epsilon p$(L=2).
Three amplitudes are $\neq 0$:$(pe^+\psi_X\psi^*)B_0, (q\psi^*)(pqe^+\psi_X)B_2, (q\psi_X)(pqe^+\psi^*)B_2$
Again,they are related,so only two amplitudes have to be considered.
They were accidentally selected as:$(pe^+\psi_X\psi^*)B_0, (q\psi_X)(pqe^+\psi^*)B_2$
Again,this is only true,when $B_0 \approx B_2$.Otherwise,all three amplitudes have to be used.

Method 4:
The multipoles $j^p = 1^-(E1)$ and $2^-(M2)$ couple with $X = 1^+$ to $J^P = 1^-$.$j^p = 1^-$ and $j^p = 2^-$ are connected to $L_{\gamma X} = 2$.
There are two independent amplitudes:
$A_1' = -\frac{M}{3}(1 - \frac{M_X^2}{M^2})(pe^+\psi_X\psi^*) \times B_2$ (E1)
$A_2' = [(1 + \frac{m_X^2}{M^2})(q\psi_X)(pqe^+\psi^*) + 2(q\psi^*)(pqe^+\psi_X)] \times B_2$ (M2).

Three terms appear:$(pe^+\psi_X\psi^*), (q\psi_X)(pqe^+\psi^*)$ and $(q\psi^*)(pqe^+\psi_X)$.Using the method of [6],App.A/B,a relation between the three terms can be established:
$\frac{3}{4}A_1' + \frac{1}{M}A_2' = 4(\vec{q}\vec{\psi_X})(\vec{q}\vec{e^+}\vec{\psi^*}) = -\frac{2}{M}(1 + \frac{M_X^2}{M^2})(q\psi_X)(pqe^+\psi^*)$
$\frac{3}{4}A_1' - \frac{1}{M}A_2' = 4(\vec{q}\vec{\psi^*})(\vec{q}\vec{e^+}\vec{\psi_X}) = -\frac{4}{M}(q\psi^*)(pqe^+\psi_X)$
As the coefficients are Lorentz-invariant,the relations hold for any arbitrary coordinate system.
The use of the linear combinations given above is thus equivalent to the use of only two of the three terms.

Resume:All four methods deliver the same terms,however in different linear combinations.

Case 4:$1^- \rightarrow 1^-(\gamma) + 1^-$

For the massive case the combinations $(L^P = 3^-; S_{Tot} = 2^+)$,$(L^P = 1^-; S_{Tot} = 2^+)$, $(L^P = 1^-; S_{Tot} = 1^+)$ and $(L^P = 1^-; S_{Tot} = 0^+)$ contribute.SP=+1.Both the LS-formalism as well as the helicity formalism deliver four amplitudes.The helicity formalism delivers two amplitudes for the radiative process.

Method 1:
Step1:The LS-amplitudes for the massive case are(Starting point were the equat.(88) of [6])
$A_1 = (\tilde{\psi_X}e)(\tilde{r}\psi^*)=$
$= 2[(e\psi_X)(q\psi^*) - \frac{1}{M^2}(q\psi^*)(q\psi_X)(ke)] \times B_1$ (L=1,S=0)
$A_2 = (\tilde{r}\psi_X)(\tilde{e}\psi^*) - (\tilde{r}e)(\tilde{\psi_X}\psi^*)=$

$= [(1+\frac{M_X^2}{M^2})(e\psi^*)(q\psi_X)+(1-\frac{M_X^2}{M^2})(ke)(\psi^*\psi_X)]\times B_1$ (L=1,S=1)
$A_3=(\tilde{r}\psi_X)(\tilde{e}\psi^*)+(\tilde{r}e)(\tilde{\psi_X\psi^*})-2/3(\tilde{\psi_X e})(\tilde{r}\psi^*)=$
$=[(1+\frac{M_X^2}{M^2})(e\psi^*)(q\psi_X)-(1-\frac{M_X^2}{M^2})(ke)(\psi^*\psi_X)-4/3[(e\psi_X)(q\psi^*)-\frac{1}{M^2}(q\psi^*)(q\psi_X)(ke)]]\times$
B_1 (L=1,S=2)
$A_4=(\tilde{r}\psi_X)(\tilde{r}\psi^*)(\tilde{r}\psi^*)-1/5(\tilde{r}r)[(\tilde{\psi_X e})(\tilde{r}\psi^*)+(\tilde{r}e)(\tilde{\psi_X\psi^*})+(\tilde{r}\psi_X)(\tilde{e}\psi^*)]=$
$=[-2(1-\frac{M_X^4}{M^4})(ke)(q\psi_x)(q\psi^*)-\frac{(M^2-M_X^2)^2}{5M^2}[2(e\psi_X)(q\psi^*)+\frac{1}{M^2}(q\psi_X)(ke)-$
$-(1-\frac{M_X^2}{M^2})(ke)(\psi^*\psi_X)+(1+\frac{M_X^2}{M^2})(e\psi^*)(q\psi_X)]]\times B_3$ (L=3,S=2)

with r=q-k and $\tilde{r}_\alpha=\tilde{g}_{\alpha\beta}r^\beta$; $\tilde{e}_\alpha=\tilde{g}_{\alpha\beta}e^\beta$;...

Step2:With the replacement $e\rightarrow e^+$ and with $M_2=0$ one obtains for the radiative case
$A_1^,=2[(e^+\psi_X)(q\psi^*)]\times B_1$
$A_2^,=(1+\frac{M_X^2}{M^2})[(e^+\psi^*)(q\psi_X)]\times B_1$
$A_3^,=[(1+\frac{M_X^2}{M^2})[(e^+\psi^*)(q\psi_X)]-4/3[(e^+\psi_X)(q\psi^*]\times B_1$
$A_4^,=-\frac{(M^2-M_X^2)^2}{5M^2}[2(e^+\psi_X)(q\psi^*)-(1+\frac{M_X^2}{M^2})(e^+\psi^*)(q\psi_X)]\times B_3$
The relations between the amplitudes are
$A_3^,=A_2^,-2/3A_1^,$
$A_4^,=-\frac{(M^2-M_X^2)^2}{5M^2}[A_1^,+A_2^,]\times B_3/B_1$
so that only two independent amplitudes remain,in case that $B_1\approx B_3$.Otherwise,all three amplitudes have to be used.

Method 2:
Step1:For the massive case four amplitudes are selected(SP=+1)
$A_1=(e\psi_X)(r\psi^*)\times B_1=2(e\psi_X)(q\psi^*)\times B_1$ (S=0,L=1)
$A_2=[(r\psi_X)(e\psi^*)-(re)(\psi_x\psi^*)]B_1=[(q\psi_X)(e\psi^*)+(ke)(\psi_x\psi^*)]\times B_1$
(S=1,L=1)
$A_3=[(r\psi_X)(e\psi^*)+(re)(\psi_x\psi^*)]B_1=[(q\psi_X)(e\psi^*)-(ke)(\psi_x\psi^*)]\times B_1$
(S=2,L=1)
$A_4=(r\psi_X)(re)(r\psi^*)\times B_3=-2(q\psi_X)(ke)(q\psi^*)\times B_3\,(S=2,L=3)$

Step2:With the replacement $e\rightarrow e^+$ and with $M_2=0$ one obtains for the radiative case
$A_1^,=2(e^+\psi_X)(q\psi^*)B_1$
$A_2^,=(r\psi_X)(e\psi^*)B_1=(q\psi_X)(e\psi^*)B_1$
$A_3^,=A_2^,$
$A_4^,=0$

Only two independent amplitudes remain.

Method 3:
The tensors,from which the amplitudes have to be built are $q, e^+, \psi, \psi_X, g_{\mu\nu}$(L=1,SP=+1) and $q, q, q, e^+, \psi, \psi_X, g_{\mu\nu}$(L=3,SP=+1).
Two amplitudes are $\neq 0$:$(e^+\psi_X)(q\psi^*)B_1$ and $(q\psi^*)(e^+\psi_X)B_1$

Method 4:
The multipoles $j^p = 1^+(M1)$ and $2^+(E2)$ couple with $X = 1^-$ to the mother particle 1^-.$j^p = 1^+$ and $j^p = 2^+$ are connected to $L_{\gamma X} = 1$.
There are two independent amplitudes with two terms:
$A_1' = 2M^2[(e^+\psi_X)(q\psi^*) - 1/2(1 + \frac{M_X}{M^2})(e^+\psi^*)(q\psi_X)] \times B_1$ (M1)
$A_2' = 1/2[(e^+\psi_X)(q\psi^*) + (1 + \frac{M_X}{M^2})(e^+\psi^*)(q\psi_X)] \times B_1$ (E2).

Resume:All four methods deliver the same terms.

Overall conclusion
All four methods deliver the right number of amplitudes with the same terms,but they appear sometimes in different linear combinations.
The most simple approaches are methods 3/4,when one is sure,which tensor combinations will contribute.

⌈There is a special situation,when J/ψ is created in high energetic e^+e^- reactions.In $e^+e^- \to \gamma^* \to \psi$,the leptons are highly relativistic,so that only spin helicities $J_z \pm 1$ are relevant(alignment),if the quantization axis(z) is chosen along the beam direction.Thus $\psi(\lambda = 0) = 0$.According to the above,that means,that the helicity of ψ is ± 1.
(See [4],page 126,eq.41a,b.Here for spin 1/2 fermions(e.g. electrons),the components of polarization are given,using the Dirac equation.Their x-and y-components are proportional to m/E,disappearing for small m/E-ratios.)
Then $\sum_{m=1,2,3} \psi_\mu(p,m)\psi_\nu^*(p,m)$ can be replaced by
$\sum_{m=1,2} \psi_\mu(p,m)\psi_\nu^*(p,m) = \delta_{\mu\nu}(\delta_{\mu 1} + \delta_{\mu 2})$ (see [2, 3]).
The latter relation can be proven by inserting the wave functions (A2).⌋

Acknowledgement
For helpful remarks I am indebted to Bingsong Zou(CCAST,Beijing).

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A Spin Wave Functions for Spin 1 Particles

For a massive spin 1 particle at rest the most general pure state is ([14],p.53)

$$|\vec{e}\rangle = \begin{pmatrix} e_x \\ e_y \\ e_z \end{pmatrix} = e_x|e_x\rangle + e_y|e_y\rangle + e_z|e_z\rangle \text{ with}$$

$$|e_x\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, |e_y\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, |e_z\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

The spin wave functions(polarization vectors) are e_x, e_y, e_z.They are in general complex quantities and have nothing to do with the components of the spin polarization vector($\vec{P}$).In contrast to $\vec{P}$,they contain all information about the spin 1 state.

The spin polarization vector for spin 1 states is

$\vec{P} = Im(\vec{e}^* \times \vec{e})$,

the alignment tensor is

$T_{ij} = \frac{3}{2}[\frac{1}{3}\delta_{ij} - Re(e_i^* e_j)]$.

⌈ These expressions have their relativistic analogues(see[14],eq.3.4.28,3.4.29).⌋

The spin wave function in a spherical basis is

$|\vec{e}\rangle = e_+|m = +1\rangle + e_-|m = -1\rangle + e_0|m = 0\rangle$

with the basis states $|m = +1\rangle, |m = -1\rangle, |m = 0\rangle$.

Here,m = 0,±1, is the spin projection to the z-axis.

The relation between the sherical and the cartesian basis states is

$|m = +1\rangle = \frac{1}{\sqrt{2}}(-|e_x\rangle - \imath|e_y\rangle)$

$|m = -1\rangle = \frac{1}{\sqrt{2}}(+|e_x\rangle - \imath|e_y\rangle)$

$|m = 0\rangle = |e_z\rangle$.

⌈$|m = 0, \pm1\rangle$ are the spin eigenvectors of the operators J^2 and J_z:

$|+1\rangle = \frac{1}{\sqrt{2}}(-|e_x\rangle - \imath|e_y\rangle) = \frac{1}{\sqrt{2}}(-\sin\theta\cos\phi - \imath\sin\theta\cos\phi) =$

$= -\frac{1}{\sqrt{2}}\sin\theta\exp\imath\phi \propto Y_1^1, ...q.e.d.$⌋

The difference between spin polarization vectors ($\vec{P}$) and the polarization vectors($\vec{e}$) is demonstrated in the following for spin 1 states with $m = \pm1, 0$:

The spin 1 polarization states at rest are given by([5],p.44;[25],p.558)

$m = \pm1 : \vec{e}_\pm = \frac{1}{\sqrt{2}}(\mp1, -\imath, 0) = \frac{1}{\sqrt{2}}(\mp1|e_x\rangle - \imath|e_y\rangle + 0|e_z\rangle)$

$m = 0 : \vec{e}_0 = (0, 0, 1) = 0|e_x\rangle + 0|e_y\rangle + 1|e_z\rangle$.

The corresponding spin polarization vectors are

$m = \pm1 : \vec{P}_\pm = (0, 0, \pm1) = 0|\vec{e}_x\rangle + 0|\vec{e}_y\rangle \pm 1|\vec{e}_z\rangle$.

$m = 0 : \vec{P}_0 = (0,0,0)$
That means,e.g.,that the polarization vector of a spin 1 particle with m=+1 has x-and y-components $\neq 0$,but the spin polarization vector has only a z-component $\neq 0$.

⌈Spin 1/2 particles behave differently[15].Here the spin polarization vector contains all information on the state and is directly related to the polarization vector.⌋

Relativistically the spin wave functions transform to
$|e\rangle = \begin{pmatrix} e^0 \\ \vec{e} \end{pmatrix}$ with $e^0(p,m) = \frac{1}{p_0}(\vec{p}\vec{e}(p,m))$ because of (pe) = 0.
From now on,only relativistic quantities(four-vectors/tensors) will be considered.

Massive spin 1 particles
Particles at rest
The spin wave function for a massive(mass M) spin 1 particle at rest is (see above)

$$e^\mu(M, m = \lambda = \pm 1) \; = \; \tfrac{\mp 1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ \pm\imath \\ 0 \end{pmatrix}; \; e^\mu(M, m = \lambda = 0) \; = \; \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \qquad \text{(A1)}$$

The spin wave functions are defined in the overall x,y,z-coordinate system like all momentum four-vectors.In the rest system the spin projection m and the helicity λ are the same.
To get the wave functions for particles with four-momentum p the appropriate Lorentz-transformations have to be applied.They differ depending,if the states are defined in the canonical or the helicity system.
Canonical system: $e^\mu(p,m) = [\overset{0}{R}\, L_z(p)\, \overset{0}{R}{}^{-1}]^\mu_\nu\, e^\nu(0,m)$
Helicity system: $e^\mu(p,\lambda) = [\overset{0}{R}\, L_z(p)]^\mu_\nu e^\nu(0,\lambda)$

The relation between helicity and canonical spin wave functions is [5]
$e^\mu(\vec{p},\lambda) = \sum_m D^1_{m\lambda}(\overset{0}{R}) e^\mu(\vec{p},m)$

For the definitions of the rotation and boosting matrices see [5] and [14],(1.2.13),(1.2.23).

Boosted particles

Particles with $\vec{p} = (0, 0, p)$ (z-axis = quantization axis = flight direction):

Canonical system:

$$e^\mu(p, m = \pm 1) \ = \ \frac{\mp 1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ \pm\imath \\ 0 \end{pmatrix}; \quad e^\mu(p, m = 0) \ = \ \begin{pmatrix} p/M \\ 0 \\ 0 \\ E/M \end{pmatrix} \qquad \text{(A2)}$$

Helicity system:

$$e^\mu(p, \lambda = \pm 1) \ = \ \frac{\mp 1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ \pm\imath \\ 0 \end{pmatrix}; \quad e^\mu(p, \lambda = 0) \ = \ \begin{pmatrix} p/M \\ 0 \\ 0 \\ E/M \end{pmatrix} \qquad \text{(A3)}$$

In this case the form of the wave functions is the same in both systems.

Particles with momentum in direction (θ, ϕ): (see [14],(1.2.23))

Canonical system:(see [7])

$$e^\mu(p, m = \mp 1) \ = \ \frac{\pm 1}{M\sqrt{2}} \begin{pmatrix} p_x \mp \imath p_y \\ M + p_x(p_x \mp \imath p_y)/(E+M) \\ \mp\imath M + p_y(p_x \mp \imath p_y)/(E+M) \\ p_z(p_x \mp \imath p_y)/(E+M) \end{pmatrix}$$

$$e^\mu(p, m = 0) \ = \ \frac{1}{M} \begin{pmatrix} p_z \\ p_z p_x/(E+M) \\ p_z p_y/(E+M) \\ M + p_z^2/(E+M) \end{pmatrix} \qquad \text{(A4)}$$

Helicity system:

$$e^\mu(p, \lambda = \pm 1) \ = \ \frac{\mp 1}{\sqrt{2}} \begin{pmatrix} 0 \\ \cos\theta\cos\phi - \imath\sin\phi \\ \cos\theta\sin\phi \pm \imath\cos\phi \\ -\sin\theta \end{pmatrix}$$

$$e^\mu(p, \lambda = 0) = \begin{pmatrix} p/M \\ E/M\sin\theta\cos\phi \\ E/M\sin\theta\sin\phi \\ E/M\cos\theta \end{pmatrix} \text{(A5)}$$

with $p_x = p\sin\theta\cos\phi; p_y = p\sin\theta\sin\phi; p_z = p\cos\theta$

The relation $(pe(\lambda))$ =0 holds for all λ. (A6)

The projection operator is

$P^{(1)}_{\mu\nu}(p) = \sum_{\lambda=\pm1,0} e_\mu(\lambda)e^*_\nu(\lambda) = -\tilde{g}_{\mu\nu}(p) = -g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2}$ (A7)
The relations (A6) and (A7) hold also for the canonical system($\lambda \to m$).

Massless spin 1 particles,e.g. photons(M = 0):
Massless particles,e.g.photons,have no rest system,so only states in the helicity basis can be defined.For particles with spin s only two states are possible:$\lambda = \pm s$.Instead of a rest state,the state (A3) serves as a standard state.
Because of the gauge invariance,which has to respected in order to reduce the number of independent wave functions to two,the spin wave functions $e^+_\mu(q,\lambda)$ and the projection operators depend on the gauge used.The choice of gauge is best demonstrated by looking at the projection operator or equivalently the electromagnetic propagator.

In the quadratic gauge(field theoretical approach) the projection operator for a photon(real and virtual) is(see [10],ch.13(here it is given as propagator))

$$P^{(1)}_{\mu\nu}(q) = \sum_{\lambda=\pm1} e^+_\mu(\lambda)e^{+*}_\nu(\lambda) = -g_{\mu\nu} + \frac{n^*_\mu q_\nu}{(qn^*)} + \frac{n_\nu q_\mu}{(qn)} - \frac{(q^2+n^*\cdot n)}{|qn|^2}q_\mu q_\nu \qquad \text{(A8)}$$

n_μ is an arbitrary four-vector,which is used to finally settle the gauge.
The expression (A8) is not quite identical to the expressions given in [11],ch.14.6 and [13],ch.15.6.This is due to the different basis vectors used.In the latter cases a term $\propto n_\mu n_\nu$ appears which is absent here.In the calculation of electromagnetic processes this term is compensated by the static Coulomb interaction in the Hamiltonian and disappears([13],p.349;[12],p.156).

Besides the quadratic gauge there is a second way to fix the gauge.It is the singular gauge fixing,used in most textbooks and papers.
The singular gauge fixing can be obtained from (A8) by replacing n by $\lambda_G n$ with $\lambda_G \to \infty$.Then the relation (A8) reads

$$P^{(1)}_{\mu\nu}(q) = \sum_{\lambda=\pm1} e^+_\mu(\lambda)e^{+*}_\nu(\lambda) = -g_{\mu\nu} + \frac{n^*_\mu q_\nu}{(qn^*)} + \frac{n_\nu q_\mu}{(qn)} - \frac{(n^* n)}{|qn|^2}q_\mu q_\nu \qquad \text{(A9)}$$

For n_μ being a real four vector (A9) reads

$$P^{(1)}_{\mu\nu}(q) = \sum_{\lambda=\pm1} e^+_\mu(\lambda)e^{+*}_\nu(\lambda) = -g_{\mu\nu} + \frac{n_\mu q_\nu + n_\nu q_\mu}{(nq)} - \frac{n^2 q_\mu q_\nu}{(nq)^2} \qquad \text{(A10)}$$

The expressions (A9/A10) are identical to the ones given in [22],p.242;[12],Ch.15.1; [21],Ch.6.4;[13]Ch.15.6;[26]Ch.3.2. for real photons($q^2 = 0$).

In the singular gauge $(ne^+) = 0$ holds.

The relations (A9)/(A10) can be also obtained by inserting the gauge invariant photon spin wave function ([6])

$e^+_\mu(q, \lambda = \pm 1) = e_\mu(q, \lambda \pm 1) + \frac{(ne(\lambda=\pm 1))}{(nq)} q_\mu,$ (A11)

into the projection operator
$P^{(1)}_{\mu\nu}(q) = \sum_{\lambda=\pm 1} e^+_\mu(\lambda) e^{+*}_\nu(\lambda) = \sum_{\lambda=0,\pm 1} \epsilon^+_\mu(\lambda) \epsilon^{+*}_\nu(\lambda)$
$(\epsilon^+_\mu(\lambda = 0) \to 0$ for $E \to |\vec{q}|)$

respecting $e^+_\mu(q, \lambda{=}0) = 0$ and exchanging the sequence of additions.
Here,$e^+_\mu(q, \lambda = \pm 1)$ is the spin wave function of the photon,$e_\mu(q, \lambda = \pm 1)$ is the spin wave function of a massive spin 1 particle.
Note: The spin wave functions for massive spin 1 particles with $\lambda = \pm 1$ are independent of the mass(see (A2),(A3)).

The use of (A9)/A10) or (A11) is equivalent.

In the following several common gauges (quadratic and singular)are presented:

<u>Covariant gauges</u>

Lorentz(better:Lorenz) gauges:Here $n_\mu \propto q_\mu : n_\mu = \imath \lambda_G q_\mu$

Insertion in (A8) yields:
$P^{(1)}_{\mu\nu}(q) = \sum_{\lambda=\pm 1} e^+_\mu(\lambda) e^{+*}_\nu(\lambda) = -g_{\mu\nu} + (1 - \lambda_G^{-2}) \frac{q_\mu q_\nu}{q^2}$

The choice of λ_G finally settles the gauge:
(1) $\lambda_G = 1$ (<u>Feynman gauge</u>,quadratic gauge)

$P^{(1)}_{\mu\nu}(q) = \sum_{\lambda=\pm 1} e^+_\mu(\lambda) e^{+*}_\nu(\lambda) = -g_{\mu\nu}$
(see also [13],p.351;[16],p.138;[11],p.156)

(2) $\lambda_G \to \infty$ (<u>Landau gauge</u>,singular gauge)

$P^{(1)}_{\mu\nu}(q) = \sum_{\lambda=\pm 1} e^+_\mu(\lambda) e^{+*}_\nu(\lambda) = -g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}$

With $(ne) = \imath\lambda_G(qe) = 0$ the spin wave function (A11) is

$e^+_\mu(\lambda = \pm1) = e_\mu(\lambda = \pm1) \quad$ with $(ne^+) = 0$ (see (A5)).

<u>Non-covariant gauges</u>

(3)$n_\mu \not\propto q_\mu$,instead $n_i \propto q_i$(three vector,i=1,2,3):

$n_i = \imath\lambda_G q_i; \qquad n_0 = 0 \qquad$ (Coulomb type gauge)

Insertion in (A8) yields (with $\mu = i, 0; \nu = j, 0; i, j = 1, 2, 3$)

$$P^{(1)}_{\mu\nu}(q) = \sum_{\lambda=\pm1} e^+_\mu(\lambda)e^{+*}_\nu(\lambda) = \begin{pmatrix} \delta_{ij} - \frac{q_iq_j}{|\vec{q}|^2} + q^2\frac{q_iq_j}{\lambda_G^2|\vec{q}|^4} & q^2\frac{q_0q_i}{\lambda_G^2|\vec{q}|^4} \\ q^2\frac{q_0q_i}{\lambda_G^2|\vec{q}|^4} & q^2(-\frac{1}{|\vec{q}|^2} + \frac{q_0^2}{\lambda_G^2|\vec{q}|^4}) \end{pmatrix}$$

For $\lambda_G \to \infty$(singular gauge) the projection operator is

$$P^{(1)}_{\mu\nu}(q) = \sum_{\lambda=\pm1} e^+_\mu(\lambda)e^{+*}_\nu(\lambda) = \begin{pmatrix} (\delta_{ij} - \frac{q_iq_j}{|\vec{q}|^2}) & 0 \\ 0 & -\frac{q^2}{|\vec{q}|^2} \end{pmatrix}$$

This gauge is known as the <u>standard Coulomb gauge</u> or <u>radiation gauge</u>.

With $(ne) = \imath\lambda_G(\vec{q}\vec{e})$ and $(nq) = \imath\lambda_G|\vec{q}|^2$ the spin wave function (A11) is

$e^+_\mu(q, \lambda = \pm1) = e_\mu(q, \lambda = \pm1) - \frac{(\vec{q}\vec{e}(\lambda=\pm1))}{|\vec{q}|^2}q_\mu \qquad$ with $(ne^+) = 0$.

(4) n is proportional to a momentum vector of the decay under consideration,e.g.,the four vector of the second decay particle (k)
$n_\mu = \lambda_G k_\mu$
In the singular gauge $\lambda_G \to \infty$ the projection operator is

$$P^{(1)}_{\mu\nu}(q) = \sum_{\lambda=\pm1} e^+_\mu(\lambda)e^{+*}_\mu(\lambda) = -g_{\mu\nu} + \frac{k_\mu q_\nu + k_\nu q_\mu}{(kq)} - \frac{n^2 q_\mu q_\nu}{(kq)^2} \qquad \text{(A12)}$$
(see [21],p.158)

This gauge is called <u>axial gauge</u>.

The spin wave function in this case is

$e^+_\mu(q, \lambda = \pm1) = e_\mu(q, \lambda \pm 1) + \frac{(ke(\lambda=\pm1))}{(kq)}q_\mu$

This spin wave function is used for the construction of the amplitudes above.

⌈If k is pointing along the time axis the gauge is called the temporal gauge (k=(1,0,0,0)),if k is pointing along the light cone the gauge is called light cone gauge(k^2 =0).⌋

In all cases note Ward's identity:When a radiative matrix element is calculated,very often(Conserved electrical current)the Ward identity holds ([13],Ch.1.1;[12],Ch.15.1;[11],p.191;[21],p.158;[26],ch.3.2),which has the effect that all terms $\propto q^\mu, q^\nu$ disappear.
The result is,that in calculations of physical quantities with (A9) or (A10) only $g_{\mu\nu}$ survives,that is

$$P^{(1)}_{\mu\nu}(q) = \sum_{\lambda=\pm1} e^+_\mu(q,\lambda) e^{+*}_\mu(q,\lambda) = -g_{\mu\nu} \qquad \text{(A13) and}$$

$$e^+(q,\lambda=\pm1) = e(q,\lambda=\pm1) = \frac{\mp1}{\sqrt{2}} \begin{pmatrix} 0 \\ \cos\theta\cos\phi - \imath\sin\phi \\ \cos\theta\sin\phi \pm \imath\cos\phi \\ -\sin\theta \end{pmatrix} \text{ (see(A5))}$$

can be commonly used.

This,however,is only true for electromagnetic processes.In strong interactions,described by the exchange of gluons(non-abelian theory),the spin wave functions and the projection operators depend on the gauge used(see above).A clever selection of gauge can facilitate the calculations considerably.The final results for physical quantities,however,must be independent of the gauge.

B Kinematical relations

For the processes $J/\psi(M,p) \to 2(M_2,q) + X(M_X,k)$ and $J/\psi(M,p) \to \gamma(0,q) + X(M_X,k)$ some useful kinematical relations are given in the following:
p,q,k = four-vectors of the particles.

Massive case($M_2 \neq 0$): $p^2 = M^2; q^2 = M_2^2; k^2 = M_X^2$

$$p = \begin{pmatrix} p^0 = \sqrt{M^2 + |\vec{p}|^2} \\ p^1 \\ p^2 \\ p^3 \end{pmatrix}; \quad q = \begin{pmatrix} q^0 = \sqrt{M_2^2 + |\vec{q}|^2} \\ q^1 \\ q^2 \\ q^3 \end{pmatrix}; \quad k = \begin{pmatrix} k^0 = \sqrt{M_X^2 + |\vec{k}|^2} \\ k^1 \\ k^2 \\ k^3 \end{pmatrix}$$

The spin wave functions are: $\psi(J/\psi)$; $e(M_2)$; $\psi_X(X)$
with $(p\psi) = (qe) = (k\psi_X) = 0$ (re) = -(ke); (r ψ_X) = (q ψ_X)
p = q + k; r = q - k = 2q - p $p^2 = M^2 = (q+k)^2$; $q^2 = M_2^2 = (p-k)^2$;
$k^2 = M_X^2 = (p-q)^2$
$(qk) = \frac{1}{2}(M^2 - M_X^2 - M_2^2) \to (qk) = \frac{1}{2}(M^2 - M_X^2)$ for $M_2 \ll M, M_X$

Photons($M_2 = 0$): $p^2 = M^2; q^2 = 0; k^2 = M_X^2$

$$p = \begin{pmatrix} p^0 = \sqrt{M^2 + |\vec{p}|^2} \\ p^1 \\ p^2 \\ p^3 \end{pmatrix}; \quad q = \begin{pmatrix} q^0 = \sqrt{|\vec{q}|^2} \\ q^1 \\ q^2 \\ q^3 \end{pmatrix}; \quad k = \begin{pmatrix} k^0 = \sqrt{M_X^2 + |\vec{k}|^2} \\ k^1 \\ k^2 \\ k^3 \end{pmatrix}$$

The spin wave functions are: $\psi(J/\psi)$; $e^+(\gamma)$; $\psi_X(X)$
with $(p\psi) = (qe^+) = (k\psi_X) = 0$ $(re^+) = -(ke^+)$; $(r\psi_X) = (q\psi_X)$
For the axial gauge the relations (pe^+)=0 and (ke^+)=0 hold in addition.

p = q + k; r = q - k = 2q - p $p^2 = M^2 = (q+k)^2$; $q^2 = 0 = (p-k)^2$;
$k^2 = M_X^2 = (p-q)^2$
$(qk) = \frac{1}{2}(M^2 - M_X^2)$

CM-system: $\vec{p} = 0$; $\vec{q} = -\vec{k}$
Massive case:

$$p = \begin{pmatrix} M \\ \vec{0} \end{pmatrix}; \quad q = \begin{pmatrix} q^0 = \sqrt{M_2^2 + |\vec{q}|^2} \\ \vec{q} \end{pmatrix}; \quad k = \begin{pmatrix} k^0 = \sqrt{M_X^2 + |\vec{q}|^2} \\ -\vec{q} \end{pmatrix}$$

$M = k^0 + q^0 \to M = k^0 + |\vec{q}|$ for very small M_2

The following relations are important for the determination of eventual relations between massive amplitudes(see [6],App.A/B).

$$(p\psi) = 0 = (p^0\psi^0 - \vec{p}\vec{\psi}) \rightarrow \psi^0 = \frac{1}{p^0}(\vec{p}\vec{\psi}) \rightarrow (q\psi) = -(\vec{q}\vec{\psi})$$
$$(k\psi_X) = 0 = (k^0\psi_X^0 - \vec{k}\vec{\psi_X}) \rightarrow \psi_X^0 = \frac{1}{k^0}(\vec{k}\vec{\psi_X}) \rightarrow (q\psi_X) = -\frac{M}{k^0}(\vec{q}\vec{\psi_X})$$
$$(qe) = 0 = (q^0e^0 - \vec{q}\vec{e}) \rightarrow e^0 = \frac{1}{q^0}(\vec{q}\vec{e}) \rightarrow (ke) = \frac{M}{q^0}(\vec{q}\vec{e})$$

C FORM-FACTORS

From the study of measured resonance shapes it becomes obvious,that a q^L dependence is not sufficient for a proper description of the amplitudes.A damping factor has to be added,which usually can only be determined using a model(see,e.g.,[27]).That is the reason,that several types of damping factors are on the market.
For strong interactions between resonances the Blatt-Weisskopf factors(finite size approach) are in common use.Usually they contain the q^L factor,which here is taken out,because it is part of the tensor amplitude.The amplitude $(qe)(pq\psi_X\psi)$,e.g.,contains a q^2-term,corresponding to L=2.For the reaction $a \to b + c$ only the truncated part of the Blatt-Weisskopf factors is given in the following,which is responsible for the damping:

$B_0 = 1$

$B_1(Q_{abc}) = \sqrt{\frac{2}{Q_{abc}^2 + Q_0^2}}$

$B_2 = \sqrt{\frac{13}{Q_{abc}^4 + 3Q_{abc}^2 Q_0^2 + 9Q_0^4}}$
with the hadron scale parameter $Q_0 = 0.197321/\mathrm{R}$ GeV/c $(R \approx 1fm)$
Q_{abc} is the magnitude of the momenta of b/c in the rest system of a and can be calculated in a covariant manner(see [2];[18],App. A4):

$Q_{abc}^2 = \frac{(s_a + s_b - s_c)^2}{4s_a} - s_b$ with $s_a = E_a^2 - \vec{p_a}^2$

The Blatt-Weisskopf factors [32] originate from a potential calculation considering an interaction of finite range(R),and are therefore adequate for the treatment of strong interaction reactions,but they are also used for electromagnetic reactions.

Other forms of form factors are also in use,e.g.,

$B \propto \exp[\frac{-q^2}{16\beta^2}]$ [33].

There are further expressions for form factors valid for special models (Veneziano,etc.)[34].

In case of electromagnetic reactions other forms may be more adequate:
(a)The vector meson dominance approach(see e.g. [17]):

$B \propto \frac{1}{1-\frac{q^2}{m_\rho^2}}$ with m_ρ being the mass of the rho-meson.

For $\gamma\gamma$ reactions,e.g.,this approach has been compared to the finite size approach(Blatt-Weisskopf) in [17],Ch. 3.1;[18],Ch.2.1.10.

(b)The CLEO-approach for E1/M1 transitions [27]

$B \propto \exp[-E_\gamma^2/E_0^2]$

This form factor is used for the description of the radiative process $J/\psi \to \gamma\eta_c$. The form originates from a complete calculation of electromagnetic processes,yielding $\Gamma \propto E_\gamma^3 \exp[-E_\gamma^2/E_0^2]$.The width is always proportional to E_γ^3[28, 29, 30] corresponding to $p_{Photon}^{2L} \times p_{Photon}$(phase space).The factor $\exp[-E_\gamma^2/E_0^2]$ with E_0=65 MeV is the damping factor,inspired by the wave function overlap.

(c)Williams approach [19]
Here, special form factors for the coupling of the nucleons to the gamma are used(see [19],p.92).

D Spin 1 Projection Operators for sherical and cartesian bases

The usual form of the spin 1 projection operator is

$\sum_{m=0,\pm1} e_\mu(p,m)e^*_\nu(p,m) = \frac{p_\mu p_\nu}{p^2} - g_{\mu\nu} = \tilde{g}_{\mu\nu}$ (D1)

The basic states are $|m=1\rangle$,$|m=-1\rangle$ and $|m=0\rangle$,and the spin wave function is
$|e(p)\rangle = \begin{pmatrix} e^0(p) \\ \vec{e}(p) \end{pmatrix}$ with
$\vec{e}(p) = e_+(p)|1\rangle + e_-(p)|-1\rangle + e_0(p)|0\rangle$ and
$e^0(p,m) = \frac{1}{p_0}(\vec{p}\vec{e}(p,m))$ because of (pe) = 0.

(D1) can be translated into a cartesian system with the basis states $|e_x\rangle = (1,0,0), |e_y\rangle = (0,1,0), |e_z\rangle = (0,0,1)$.
The spin wave function is

$|e(p)\rangle = e_x(p)|e_x\rangle + e_y(p)|e_y\rangle + e_z(p)|e^z\rangle.$

For spin 1 particles the relations between sperical states $(|+1\rangle, |-1\rangle, |0\rangle)$ and cartesian states $(|e^x\rangle, |e^y\rangle, |e^z\rangle)$ are given in App.A.

Thus the relation between the spin wave components in the spherical and the cartesian basis is:
$e_+(p) = \frac{1}{\sqrt{2}}(-e_x(p) - \imath e_y(p))$
$e_-(p) = \frac{1}{\sqrt{2}}(+e_x(p) - \imath e_y(p))$
$e_0(p) = e_z(p)$ (D3)

For the case of a particle with $p = (E,0,0,p_z = p)$,e.g.,the spherical wave functions (A3) transform into

$$e_x(p) = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}; \quad e_y(p) = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}; \quad e_z(p) = \begin{pmatrix} p/M \\ 0 \\ 0 \\ E/M \end{pmatrix} \qquad \text{(D4)}$$

The relation
$\sum_{m=0,\pm1} e_\mu(p,m)e^*_\nu(p,m) = \sum_{m=x,y,z} e_m(p)e^*_m(p) = \frac{p_\mu p_\nu}{p^2} - g_{\mu\nu} = \tilde{g}_{\mu\nu}$

can be proven either for the special case above by using (D4) and (A3) or more generally by using (D3):

$\sum_{m=0,\pm 1} e_\mu(p,m) e^*_\nu(p,m) =$
$=1/2((-e_x(p) - \imath e_y(p))(-e^*_x(p) + \imath e^*_y(p)) +$
$+(+e_x(p) - \imath e_y(p))(+e^*_x(p) + \imath e^*_y(p))) +$
$+e_z(p) e^*_z(p) =$
$=\sum_{m=x.y.z} e_m(p) e^*_m(p),$ q.e.d.

The same is true for $m \rightarrow \lambda$(canonical to helicity formalism).Then

$\sum_{m=0,\pm 1} e_\mu(p,m) e^*_\nu(p,m) = \sum_{\lambda=0,\pm 1} e_\mu(p,\lambda) e^*_\nu(p,\lambda) = \sum_{m=x,y,z} e_m(p) e^*_m(p) =$

$= \frac{p_\mu p_\nu}{p^2} - g_{\mu\nu} = \tilde{g}_{\mu\nu}$ (D5)

This equivalence can be easily checked,when the flight direction of the particle is in z-direction(m=λ),or more generally by inserting the explicit spin wave functions from App.A in (D5).