Helicity Amplitude for $\overline{p}p \to \omega\pi^0, \omega \to \pi^0\gamma$,v3

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Abstract

The analysis of $\overline{p}$ p reactions in flight has been started up to now from the J^{PC} intermediate states(Standard Method).The problem with the Standard Method is,that additional Clebsch-Gordan Coefficients describing the coupling of the $\overline{p}$ p system with the J^{PC} system are not taken correctly into account.The same is true for the kinematical factors(different for different beam-energies).They are important in the comparison of amplitudes at different $\overline{p}$ energies.Here,the whole reaction chain is described starting from the antiproton- and proton- helicity states,avoiding the above mentioned complications.That is done for the $\overline{p}p \to \omega\pi^0, \omega \to \pi^0\gamma$ reaction,but can be easily expanded to more general cases.A comparison of the results with the Standard Method is performed showing that the Standard Method does not reproduce the correct sign of the amplitudes.Also the spin density matrix of the omega is discussed and the determination of its elements using different methods is outlined..

1 Definitions for $\overline{p}p \to \omega\pi^0, \omega \to \pi^0\gamma$

The definitions of the relevant quantities for the reaction are given in Fig.1.

1.1 Quantum numbers of particles

The quantum numbers of the particles relevant for the reaction under discussion are summarized in Table 1.

1.2 Quantum numbers of Sub-Systems

12-System:
$S_{12} = 0, 1;\ L_{12} = 0, 1, 2, 3, , , ;\ P_{12} = (-1)^{L_{12}+1}$
$S_{12} = 0 : J_{12} = L_{12} = 0, 1, 2, ,$
$S_{12} = 1 : J_{12} = L_{12} \pm 1, L_{12}, = 0, 1, 2 \ldots$

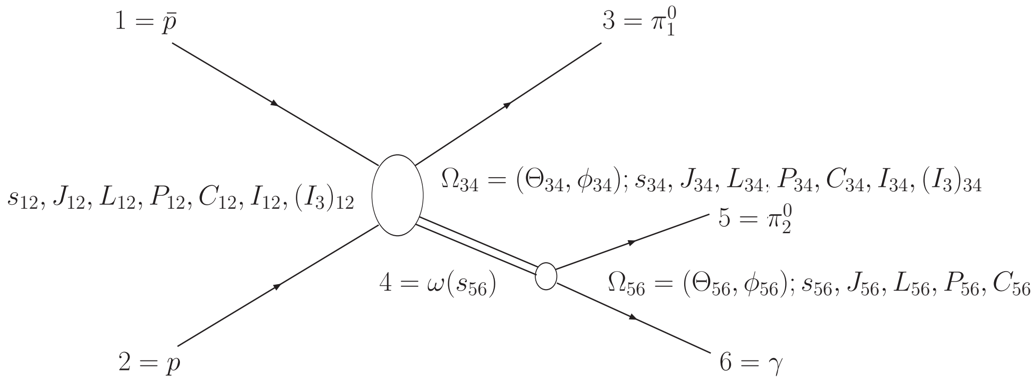

Figure 1: Definitions for the reaction $\overline{p}p \to \omega\pi^0, \omega \to \pi^0\gamma$.The helicity angles θ_{34}, Φ_{34} are measured in the overall CM-system with $\vec{z} \| \vec{p_{\overline{p}}}$.The helicity angles Θ_{56}, Φ_{56} are measured in the ω rest system,with $\vec{z} \| \vec{p_\omega}$.

Properties	1=$\overline{p}$	2=p	3=π_1^0	$4 = \omega$	5=π_2^0	$6 = \gamma$
Rest mass	$m_{\overline{p}}$	m_p	m_{π^0}	m_ω	m_{π^0}	$m_\gamma = 0$
Momentum	$\vec{p_{\overline{p}}}$	$\vec{p_p}$	$\vec{p}_{\pi_1^0}$	$\vec{p_\omega}$	$\vec{p}_{\pi_2^0}$	$\vec{p_\gamma}$
Spin	1/2	1/2	0	1	0	1
Helicities	$\pm 1/2$	$\pm 1/2$	0	$\pm 1, 0$	0	± 1
Isospin I	1/2	1/2	1	0	1	0,1
I_3	-1/2	1/2	0	0	0	-
Parity	-1	+1	-1	-1	-1	-1
C-Parity	-	-	+1	-1	+1	-1
G-Parity	-	-	-1	-1	-1	-

Table 1: Properties of the contributing particles

$C_{12} = (-1)^{L_{12}+S_{12}}$
$I_{12} = 0, 1; (I_3)_{12} = 0$
$G_{12} = (-1)^{L_{12}+S_{12}+I_{12}}$

34-System:
$S_{34} = S_\omega = 1; L_{34} = 0, 1, 2, , , ; P_{34} = (-1)^{L_{34}}$
$J_{34} = L_{34} \pm 1, L_{34}, = 0, 1, 2, , ,$
$C_{34} = -1$
$I_{34} = 1$
$G_{34} = 1$

56-System:
$S_{56} = 1; L_{56} = 1$(Parity Conservation);$P_{56} = (-1)^{L_{56}} = P_\omega = -1$
$J_{56} = J_\omega = 1$

$I_{56} = 0, 1$

1.3 Conserved Quantities

$J_{12} = J_{34} = J; J_\omega = 1 = J_{56}; P_{12} = P_{34} = P; C_{12} = C_{34} = C(= -1); P_{56} = P_\omega = -1$
$I_{12} = I_{34} = 1; (I_3)_{12} = (I_3)_{34}$ (12 → 34 transition:Strong interaction)
$(I_3)_{34} = (I_3)_{56}$ (34 → 56 transitition:Electromagnetic interaction)

1.4 CM-System

$\vec{p}_p = -\vec{p}_{\overline{p}}; |\vec{p}_{\overline{p}}| = |\vec{p}_p| = p_{\overline{p}} = p_{12}$
$\vec{p}_{\pi_1^0} = -\vec{p}_\omega; \left|\vec{p}_{\pi_1^0}\right| = |\vec{p}_\omega| = p_\omega = p_{34}$
$s_{12} = (\underline{p_1} + \underline{p_2})^2 = m_{\overline{p}}{}^2 + m_p{}^2 + 2E_{\overline{p}}E_p + 2p_{\overline{p}}{}^2 = (E_{\overline{p}} + E_p)^2$
($\underline{p}$ means four-vector)
$s_{34} = s_{12}$
$s_{56} = m_\omega^2$

2 Differential Cross Section

2.1 Differential cross section for unpolarized particles

For unpolarized antiprotons and protons the number of particles(d^5N) scattered into a phase space volume dLips is given by [1], [2]

$$d^5N \;=\; N_{initial} \times n_0 \times \Delta z \times d^5\sigma \;=\; L_{int} \times d^5\sigma \quad (n_0 = \rho_{Mol} \times A_{Av}) \tag{1}$$

with

$$\begin{aligned} d^5\sigma \;&=\; flux \times \overline{\left|T_{fi}\right|^2} \times PhaseSpace = \\ &= \frac{1}{4s^{1/2}p_{\overline{p}}} \overline{\Big|\sum_{\lambda_4} a_{iso}\, T_{\lambda_1,\lambda_2,\lambda_3\lambda_4}(\overline{p}p \to \pi_1^0\omega)\, A_{\lambda_4,\lambda_5\lambda_6}(\omega \to \pi_2^0\gamma)\Big|^2} \times \\ &\times \frac{dLips(s, p_3, p_5, p_6)}{(m_\omega^2 - s_\omega)^2 + m_\omega^2{\Gamma_\omega}^2} = \\ &= \frac{1}{4s^{1/2}p_{\overline{p}}} \times \frac{1}{2S_1+1}\frac{1}{2S_2+1} \sum_{\lambda_1,\lambda_2,\lambda_3,\lambda_5,\lambda_6} \Big|\sum_{\lambda_4} a_{iso}\, T_{\lambda_1,\lambda_2,\lambda_3\lambda_4}(\overline{p}p \to \pi_1^0\omega) \times \\ &\times A_{\lambda_4,\lambda_5\lambda_6}(\omega \to \pi_2^0\gamma)\Big|^2 \times \frac{dLips(s, p_3, p_5, p_6)}{(m_\omega^2 - s_\omega)^2 + m_\omega^2{\Gamma_\omega}^2} \qquad (\lambda_3 = \lambda_5 = 0) \end{aligned} \tag{2}$$

and

$$a_{iso} \; = \; \text{isospin dependent factor}$$

$$dLips(s,p_3,p_5,p_6) \; = \; \frac{ds_\omega}{2\pi} \times dLips(s,p_3,p_4) \times dLips(s_\omega,p_5,p_6) \qquad (3)$$

$$dLips(s,p_3,p_4) \; = \; \frac{p_{34}}{16\pi^2\sqrt{s}} \times d\Omega_{34} \; = Kin_{34} \times d\Omega_{34} \; = \; Kin_{34} \times d\cos\Theta_{34}\, d\Phi_{34} \quad (4)$$

$$dLips(s_\omega,p_5,p_6) \; = \; \frac{p_{56}}{16\pi^2\sqrt{s_\omega}} \times d\Omega_{56} \; = \; Kin_{56} \times d\Omega_{56} \; = \; Kin_{56} \times d\cos\Theta_{56}\, d\Phi_{56} \qquad (5)$$

$$\overline{|T|}^2 = \frac{1}{2S_1+1}\frac{1}{2S_2+1}\sum_{\lambda_1,\lambda_2,\lambda_3,\lambda_4} \left|T_{\lambda_1\lambda_2\lambda_3\lambda_4}\right|^2 \qquad (6)$$

The amplitudes T_{fi} are defined according to the usual conventions [2, 3, 1, 4]

For the process 1+2⟶3+4: $\frac{d\sigma}{d\Omega} = \frac{1}{64\,\pi^2\,s} \times \frac{p_{34}}{p_{12}} \times \left|T_{fi}\right|^2$
(Dimension of $T_{fi} = Dim[T_{fi}] = 1$)

For the decay $4 \rightarrow 5+6$: $\Gamma = \frac{1}{2m_4}\int \left|A_{fi}\right|^2 \times \frac{p_{56} d\Omega_{56}}{16\pi^2 m_4}$
(Dimension of $A_{fi} = Dim[A_{fi}]$=GeV)

Note:For polarized particles in the initial state $(1+2 \rightarrow 3+4)$ $\overline{\left|T_{fi}\right|}^2$has to be replaced by

$$\sum_{\lambda_1,\lambda_2,\lambda_1\prime,\lambda_2\prime} \rho^1_{\lambda_1,\lambda_1\prime}\rho^2_{\lambda_2,\lambda_2\prime} \sum_{\lambda_3,\lambda_4} T_{\lambda_1,\lambda_2,\lambda_3,\lambda_4} \times T^*_{\lambda_1\prime,\lambda_2\prime,\lambda_3,\lambda_4}$$

with the spin density matrices $\rho^1,\ \rho^2$ describing the polarization states of the initial particles.

2.2 Isospin Factor

$a_{iso} = \langle I_3, I3_3, I_4, I3_4 \mid I_1, I3_1, I_2, I3_2\rangle$ is the isospin-dependent part of the amplitude.

It can be expanded into isospin states of definite total isospin I^2 and total I3:

$$\langle I_3, I3_3, I_4, I3_4 \mid I_1, I3_1, I_2, I3_2\rangle =$$
$$= \sum_{I_{12},I3_{12},I_{34},I3_{34}} \langle I_3, I3_3, I_4, I3_4 \mid I_{34}, I3_{34}\rangle \langle I_{34}, I3_{34} \mid I_{12}, I3_{12}\rangle \langle I_{12}, I3_{12} \mid I_1, I3_1, I_2, I3_2\rangle =$$
$$= \sum_{I_{12},I3_{12}} \langle I_3, I3_3, I_4, I3_4 \mid I_{12}, I3_{12}\rangle\langle I_{12}, I3_{12} \mid I_1, I3_1, I_2, I3_2\rangle \qquad (7)$$

with

$$\langle I_{34}, I3_{34} \mid I_{12}, I3_{12} \rangle = \delta_{I_{12} I_{34}} \, \delta_{I3_{12} I3_{34}} \tag{8}$$

Nomenclature for Clebsch-Gordan's: $\langle j_1, j3_1, j_2, j3_2 \big| J, J3 \rangle$

Here: $I_{12} = 0, 1; I_{34} = 1; I3_{12} = 0; I3_{34} = 0$

$$\begin{aligned} a_{iso} &= \langle I_3, I3_3, I_4, I3_4 \mid I_1, I3_1, I_2, I3_2 \rangle = \\ &= \langle 1/2, -1/2, 1/2, 1/2 \mid 00 \rangle \times \underbrace{\langle 0, 0 \mid 1, 0, 0, 0 \rangle}_{0} + \\ &+ \langle 1/2, -1/2, 1/2, 1/2 \mid 1, 0 \rangle \times \underbrace{\langle 1, 0 \mid 1, 0, 0, 0 \rangle}_{1} = 1/\sqrt{2} \end{aligned} \tag{9}$$

Here,particles 3 and 4 are C-Parity Eigenstates,so that $C_{34} = -1 = C_{12}$. There are,however,cases,e.g $\overline{p}p \rightarrow K\overline{K}$,where states have to be constructed,which are eigenstates of I and C(G)[5].

2.3 Differential cross section for vanishing omega width

With

$$\lim_{m_\omega \Gamma_\omega \to 0} \frac{\frac{m_\omega \Gamma_\omega}{\pi}}{(m_\omega^2 - s_\omega)^2 + m_\omega^2 {\Gamma_\omega}^2} = \delta(m_{omega}^2 - s_\omega)$$

(2) can be written

$$\begin{aligned} d^5\sigma &= a_{iso}^2 \times \frac{1}{4 s^{1/2} p_{\overline{p}}} \overline{\left| \sum_{\lambda_4} T_{\lambda_1,\lambda_2,\lambda_3 \lambda_4}(\overline{p}p \rightarrow \pi_1^0 \omega) \, A_{\lambda_4,\lambda_5 \lambda_6}(\omega \rightarrow \pi_2^0 \gamma) \right|^2} \times \\ &\times \frac{\pi}{m_\omega \Gamma_\omega} \delta(m_{omega}^2 - s_\omega) \frac{ds_\omega}{2\pi} \, dLips(s, p_3, p_4) \, dLips(s_\omega, p_5, p_6) \end{aligned} \tag{10}$$

yielding

$$\begin{aligned} d^4\sigma &= \int \frac{d^5\sigma}{ds_\omega} ds_\omega = a_{iso}^2 \times \frac{1}{4 s^{1/2} p_{\overline{p}}} \frac{1}{2 m_\omega \Gamma_\omega} \overline{\left| \sum_{\lambda_4} T_{\lambda_1,\lambda_2,\lambda_3 \lambda_4}(\overline{p}p \rightarrow \pi_1^0 \omega) \, A_{\lambda_4,\lambda_5 \lambda_6}(\omega \rightarrow \pi_2^0 \gamma) \right|^2} \times \\ &\times dLips(s, p_3, p_4) \, dLips(s_\omega, p_5, p_6) \\ &= 1/4 \times a_{iso}^2 \times \frac{1}{4 s^{1/2} p_{\overline{p}}} \frac{1}{2 m_\omega \Gamma_\omega} \times \sum_{\lambda_1,\lambda_2,\lambda_3,\lambda_5,\lambda_6} \left| \sum_{\lambda_4} T_{\lambda_1,\lambda_2,\lambda_3 \lambda_4}(\overline{p}p \rightarrow \pi_1^0 \omega) \, A_{\lambda_4,\lambda_5 \lambda_6}(\omega \rightarrow \pi_2^0 \gamma) \right|^2 \times \\ &\times dLips(s, p_3, p_4) \, dLips(s_\omega, p_5, p_6) \end{aligned} \tag{11}$$

2.4 Expansion into partial waves

The amplitudes $T_{\lambda_1,\lambda_2,\lambda_3\lambda_4}(\overline{p}p \to \pi_1^0\omega)$ and $A_{\lambda_4,\lambda_5\lambda_6}(\omega \to \pi_2^0\gamma)$ are expanded into partial waves [8] with J=$J_{12} = J_{34}$ and $J_{56} = J_\omega(= 1)$

$$
\begin{aligned}
&\left|\sum_{\lambda_4} T_{\lambda_1,\lambda_2,\lambda_3\lambda_4}(\overline{p}p \to \pi_1^0\omega)\, A_{\lambda_4,\lambda_5\lambda_6}(\omega \to \pi_2^0\gamma)\right|^2 = \\
&\quad = \Big|\sum_{\lambda_4} \frac{8\pi s^{1/2}}{\sqrt{p_{\overline{p}}p_\omega}} \sum_J (2J+1) D^{J*}_{\lambda_1-\lambda_2,\lambda_4}(\Omega_{34}) \times \\
&\quad \times\langle\lambda_4, 0 \mid T^J \mid \lambda_1, \lambda_2\rangle \times \sqrt{\frac{2S_\omega+1}{4\pi}} D^{1*}_{\lambda_4\lambda_5-\lambda_6}(\Omega_{56}) \times A^1_{\lambda_5,\lambda_6}\Big|^2 = \\
&\quad = \frac{48\,\pi s}{p_{\overline{p}}p_\omega} \Big|\sum_J (2J+1) \sum_{\lambda_4} D^{J*}_{\lambda_1-\lambda_2,\lambda_4}(\Omega_{34}) D^{1*}_{\lambda_4\lambda_5-\lambda_6}(\Omega_{56}) \times \\
&\quad \times\langle\lambda_4, 0 \mid T^J \mid \lambda_1, \lambda_2\rangle \times A^1_{\lambda_5,\lambda_6}\Big|^2 \quad (J = J_{12} = J_{34}; \lambda_3 = 0, \lambda_5 = 0)
\end{aligned}
\tag{12}
$$

with the production amplitude

$$
T_{\lambda_1,\lambda_2,\lambda_3\lambda_4}(\overline{p}p \to \pi_1^0\omega) \;=\; \sum_J \frac{8\,\pi\, s^{1/2}}{\sqrt{p_{\overline{p}}p_\omega}}\,(2J+1) D^{J*}_{\lambda_1-\lambda_2,\lambda_4}(\Omega_{34}) \times \langle\, \lambda_4, 0 \mid T^J \mid \lambda_1, \lambda_2\rangle
\tag{13}
$$

(Dim[$T_{\lambda_1,\lambda_2,\lambda_3\lambda_4}(\overline{p}p \to \pi_1^0\omega)$]=Dim[$\langle\, \lambda_4, 0 \mid T^J \mid \lambda_1, \lambda_2\rangle$]= 1)

In case of parity conservation

$$
\langle\lambda_4, 0 \mid T^J \mid \lambda_1, \lambda_2\rangle = \eta_{\overline{p}} \times \eta_p \times \eta_\omega \times \eta_{\pi^0} \times (-1)^{S_\omega + S_{\pi^0} + S_{\overline{p}} + S_p} \times \langle -\lambda_4, 0 \mid T^J \mid -\lambda_1, -\lambda_2\rangle
\tag{14}
$$

The decay amplitude becomes

$$
A^{J=1}_{\lambda_4,\lambda_5\lambda_6}(\omega \to \pi_2^0\gamma) \;=\; \sqrt{\frac{2S_\omega+1}{4\,\pi}} D^{1*}_{\lambda_4\lambda_5-\lambda_6}(\Omega_{56})\, A^1_{\lambda_5,\lambda_6}
\tag{15}
$$

(Dim[$A^{J=1}_{\lambda_4,\lambda_5\lambda_6}(\omega \to \pi_2^0\gamma)$] = Dim[$A^1_{\lambda_5,\lambda_6}$] = GeV)

Parity conservation means here $A_{\lambda_5\lambda_6} \;=\; \eta_\omega\eta_\pi\eta_\gamma \times (-1)^{0+1-1} A_{-\lambda_5-\lambda_6} = -A_{-\lambda_5-\lambda_6}$

For taking into account C-conservation there is no direct way in the helicity formalism.An expension in partial waves (see later) is mandatory.

The definition of the partial wave amplitudes corresponds to the usual conventions [6, 7, 8]:

For the reaction $1+2 \rightarrow 3+4$:

$\frac{d\sigma}{d\Omega_{34}} = \frac{1}{p_{12}^2} \left| \sum_J (2J+1) D^{J*}_{\lambda_2-\lambda_1,\lambda_4-\lambda_3} \langle \lambda_3, \lambda_4 \mid T^J \mid \lambda_1, \lambda_2 \rangle \right|^2$

For the decay 4→ 5+6:

$\Gamma = \frac{1}{2m_4} \frac{p_{56}}{16\,\pi^2\,m_4} \times \sum_{\lambda_4\lambda_5\lambda_6} \int \left| A^{J=1}_{\lambda_4,\lambda_5\lambda_6}(\omega \rightarrow \pi^0_2\gamma) \right|^2 d\Omega_{56} =$
$\frac{1}{2m_4} \frac{p_{56}}{16\,\pi^2\,m_4} \times \sum_{\lambda_4\lambda_5\lambda_6} \int \left| \sqrt{3/4\pi} D^{1*}_{\lambda_4\lambda_6-\lambda_5} A^1_{\lambda_5,\lambda_6} \right|^2 d\Omega_{56} \; = \; \frac{p_{56}}{32\,\pi^2\,m_4^2} \sum_{\lambda_5\lambda_6} \left| A^1_{\lambda_5,\lambda_6} \right|^2$
using the orthogonality relation $\int \left| D^J \right|^2 \times d\Omega_{56} = \frac{4\pi}{2J+1}$

Taking into account the expansions, $d^4\sigma$ becomes

$$\begin{aligned} d^4\sigma \; &= a_{iso}^2 \, \frac{3\sqrt{s}\pi}{2p_{\bar{p}}^2 p_\omega m_\omega \, \Gamma_\omega} \times \\ &\times \sum_{\lambda_1,\lambda_2,\lambda_3,\lambda_5,\lambda_6} \left| \sum_{\lambda_4} \sum_J (2J+1) D^{J*}_{\lambda_1-\lambda_2,\lambda_4}(\Omega_{34}) \times \langle\, \lambda_4, 0 \mid T^J \mid \lambda_1, \lambda_2 \rangle \, D^{1*}_{\lambda_4\lambda_5-\lambda_6}(\Omega_{56}) \, A^1_{\lambda_5,\lambda_6} \right|^2 \times \\ &\times dLips(s, p_3, p_4) \, dLips(s_\omega, p_5, p_6) \end{aligned} \qquad (16)$$

2.5 Expansion in LS-Basis

The partial wave amplitudes $A^1_{\lambda_5,\lambda_6}$ and $\langle \lambda_4, 0 \mid T^J \mid \lambda_1, \lambda_2 \rangle$ are expanded in their LS-basis:

$\omega \rightarrow \pi^{\mathbf{0}}\,\gamma$ ($L_{56} = 1; S_{56} = 1$;only one term because of parity conservation):

$$\begin{aligned} A^1_{\lambda_5,\lambda_6} &= \sqrt{\frac{2L_{56}+1}{2S_\omega+1}} \underbrace{\langle 1, 0, 1, \lambda_6 \mid 1, \lambda_6 \rangle}_{-\lambda_6\sqrt{1/2}} \underbrace{\langle 0, 0, 1, \lambda_6 \mid 1, \lambda_6 \rangle}_{1} \times \alpha^1_{L_{56}=1,S_{56}=1} \\ &= -\frac{1}{\sqrt{2}} \times \lambda_6 \times \alpha^1_{11} \qquad (\alpha^1_{11} = \text{complex const.}) \end{aligned} \qquad (17)$$

(Dim[α^1_{11}]=GeV)

$\overline{\mathbf{p}}\mathbf{p} \rightarrow \omega\, \pi^{\mathbf{0}}$:

$$\langle \lambda_4, 0 \mid T^J \mid \lambda_1, \lambda_2 \rangle = \sum_{L_{34}, S_{34}(=1)} \sqrt{\frac{2L_{34}+1}{2J+1}} \langle L_{34}, 0, 1, \lambda_4 \mid J, \lambda_4 \rangle \underbrace{\langle 1, \lambda_4, 0, 0 \mid 1, \lambda_4 \rangle}_{1} \times$$

$$\times \sum_{L_{12}, S_{12}} \sqrt{\frac{2L_{12}+1}{2J+1}} \langle L_{12}, 0, S_{12}, \lambda_1 - \lambda_2 \mid J, \lambda_1 - \lambda_2 \rangle \langle 1/2, \lambda_1, 1/2, \lambda_2 \mid S_{12}, \lambda_1 - \lambda_2 \rangle \times$$

$$\times \langle\, {}^{2L_{34}+1}L_{34_J} \mid T^J \mid {}^{2L_{12}+1}L_{12_J} \rangle$$

$$= \sum_{L_{12}, S_{12}, L_{34}, S_{34}(=1)} \sqrt{\frac{2L_{12}+1}{2J+1}} \sqrt{\frac{2L_{34}+1}{2J+1}} \langle L_{34}, 0, 1, \lambda_4 \mid J, \lambda_4 \rangle \langle L_{12}, 0, S_{12}, \lambda_1 - \lambda_2 \mid J, \lambda_1 - \lambda_2 \rangle \times$$

$$\times \langle 1/2, \lambda_1, 1/2, -\lambda_2 \mid S_{12}, \lambda_1 - \lambda_2 \rangle \times \langle\, {}^{2L_{34}+1}L_{34_J} \mid T^J \mid {}^{2L_{12}+1}L_{12_J} \rangle \tag{18}$$

(Dim[$\langle\, {}^{2L_{34}+1}L_{34_J} \mid T^J \mid {}^{2L_{12}+1}L_{12_J} \rangle$=1)

For a fixed J-value,there exist the following L_{12}, S_{12} combinations:

For J=0:
$S_{12} = 0: \; L_{12} = 0$
$S_{12} = 1: \; L_{12} = 1$

For J≥1:
$S_{12} = 0: \; L_{12} = J$
$S_{12} = 1: \; L_{12} = J-1, J, J+1$

In our case,for a given J there exist twelve $(3 \times 2 \times 2)$ independent $\langle \lambda_4, 0 \mid T^J \mid \lambda_1, \lambda_2 \rangle$ -amplitudes.Their number is reduced to four(two),using Parity-and C-Parity-conservation.
Because of Parity-and C-Parity conservation eight of the twelfe amplitudes are zero:
Parity conservation:$P_{12} = (-1)^{L_{12}+1} = P_{34} = (-1)^{L_{34}}$
C-Parity conservation:$C_{12} = (-1)^{L_{12}+S_{12}} = C_{34} = -1$

An example for J=1 is given in Appendix A.

For even J only two partial waves remain(see also [9])
Reason:Because of $(-1)^{L_{12}+S_{12}} = C_{34} = -1, \qquad L_{12} + S_{12}$ must be odd.
$S_{12} = 0: L_{12} = J_{12} = odd \longrightarrow$ Term contributes only for $J_{12} = odd$
$S_{12} = 1: L_{12} = J_{12} - 1, J_{12}, J_{12} + 1 = even \longrightarrow$ Term contributes once for J_{12}=even,two times for J_{12}=odd

A special case is J=0.Here,only ${}^3P_0$ and ${}^1S_0$ can contribute.Both have C=+1,so that in our case(C=-1) the J=0-amplitudes are zero.

2.6 Final Amplitude

From (11,3,4,12,17,18)the final expression for the cross section is derived:

$$\frac{d^4\sigma}{d\cos\Theta_{34}d\cos\Theta_{56}d\Phi_{34}d\Phi_{56}} = a_{iso}^2 \frac{3\,p_\gamma}{512\,\pi^3\,p_{\bar{p}}^2\,m_\omega^2\,\Gamma_\omega} \times$$
$$\times \sum_{\lambda_1,\lambda_2,\lambda_6} \Big| \sum_J (2J+1) \sum_{\lambda_4} D^{J*}_{\lambda_1-\lambda_2,\lambda_4}(\Omega_{34})\, D^{1*}_{\lambda_4\lambda_5-\lambda_6}(\Omega_{56}) \times$$
$$\times \sum_{L_{12},S_{12},L_{34},S_{34}(=1)} \sqrt{\frac{2L_{12}+1}{2J+1}} \sqrt{\frac{2L_{34}+1}{2J+1}} \langle L_{34},0,1,\lambda_4 \mid J,\lambda_4\rangle \langle L_{12},0,S,\lambda_1-\lambda_2 \mid J,\lambda_1-\lambda_2\rangle \times$$
$$\times \langle 1/2,\lambda_1,1/2,-\lambda_2 \mid S,\lambda_1-\lambda_2\rangle \times \lambda_6 \times \hat{T}^{J^{PC}}_{L_{12},S_{12},L_{34},S_{34}(=1)} \Big|^2 \qquad (19)$$

with the complex Fit Parameters

$$\hat{T}^{J^{PC}}_{L_{12},S_{12},L_{34},S_{34}(=1)} = \langle {}^{2S_{34}+1}L_{34_J} \big| T^J \big| {}^{2S_{12}+1}L_{12_J}\rangle \times \alpha^1_{11}$$

(Dim[$\hat{T}^{J^{PC}}_{L_{12},S_{12},L_{34},S_{34}(=1)}$]=GeV)

$\hat{T}^{J^{PC}}_{L_{12},S_{12},L_{34},S_{34}(=1)} \neq 0$ only for
$P_{12} = (-1)^{L_{12}+1} = P_{34} = (-1)^{L_{34}}$(Parity conservation) and
$C_{12} = (-1)^{L_{12}+S_{12}} = C_{34} = -1$(C-Parity conservation)

The number of fit parameters(4 complex numbers for J odd and two complex numbers for J even)increases with s (see [9, 10, 11])

The angle Φ_{34} is not defined(except for a polarized initial state),therefore d$^4\sigma$ has to be integrated over Φ_{34} (see Appendix B).
The product $d\cos\Theta_{34}\,d\cos\Theta_{56}\,d\Phi_{34}\,d\Phi_{56}$ can be written in terms of the angle between production and decay plane $\Phi_{56'} = \Phi_{56} - \Phi_{34}$

$$d\cos\Theta_{34}\,d\cos\Theta_{56}\,d\Phi_{34}\,d\Phi_{56} = d\cos\Theta_{34}\,d\cos\Theta_{56}\,d\Phi_{34}\,d\Phi_{56'} = d\Omega_{34}d\Omega_{56'} \quad (20)$$

with

$$d\Omega_{56'} = d\cos\Theta_{56}\,d\Phi_{56'}$$

The 3-dimensional differential cross section is:

$$\frac{d^3\sigma}{d\cos\Theta_{34} d\cos\Theta_{56} d\Phi_{56'}} = \int \frac{d^4\sigma}{d\cos\Theta_{34} d\cos\Theta_{56} d\Phi_{34} d\Phi_{56'}} d\Phi_{34} =$$
$$= 2\pi \times a_{iso}^2 \times \frac{3\,p_\gamma}{512\,\pi^3\,p_{\overline{p}}^2\,m_\omega^2\,\Gamma_\omega} \times \sum_{\lambda_1,\lambda_2,\lambda_6} \Big| \sum_J (2J+1) \sum_{\lambda_4} d^J_{\lambda_1-\lambda_2,\lambda_4}(\Theta_{34})\, D^{1*}_{\lambda_4\lambda_5-\lambda_6}(\Omega_{56'}) \times$$
$$\times \sum_{L_{12},S_{12},L_{34},S_{34}(=1)} \sqrt{\frac{2L_{12}+1}{2J+1}} \sqrt{\frac{2L_{34}+1}{2J+1}} \langle L_{34},0,1,\lambda_4 \mid J,\lambda_4\rangle \langle L_{12},0,S_{12},\lambda_1-\lambda_2 \mid J,\lambda_1-\lambda_2\rangle \times$$
$$\times \langle 1/2,\lambda_1,1/2,-\lambda_2 \mid S_{12},\lambda_1-\lambda_2\rangle \times \lambda_6 \times \hat{T}^{J^{PC}}_{L_{12},S_{12},L_{34},S_{34}(=1)} \Big|^2 =$$
$$= 2\pi \times a_{iso}^2 \times \frac{3\,p_\gamma}{512\,\pi^3\,p_{\overline{p}}^2\,m_\omega^2\,\Gamma_\omega} \times w \tag{21}$$

with w = weight of the event.

For J=1,e.g.,the expression reads:

$$\frac{d^3\sigma}{d\cos\Theta_{34} d\cos\Theta_{56} d\Phi_{56'}} = 2\pi \times a_{iso}^2 \times \frac{3\,p_\gamma}{512\,\pi^3\,p_{\overline{p}}^2\,m_\omega^2\,\Gamma_\omega} \times$$
$$\times \sum_{\lambda_1,\lambda_2,\lambda_6} \times \Big| ....(2\times 1+1) \sum_{\lambda_4} D^{J*}_{\lambda_1-\lambda_2,\lambda_4}(\Omega_{34})\, D^{1*}_{\lambda_4\lambda_5-\lambda_6}(\Omega_{56'}) \times$$
$$\times [\sqrt{1/3} \times \langle 0,0,1,\lambda_4 \mid 1,\lambda_4\rangle \times \sqrt{3/3}\langle 1,0,0,\lambda_1-\lambda_2 \mid 1,\lambda_1-\lambda_2\rangle \times$$
$$\times \langle 1/2,\lambda_1,1/2,-\lambda_2 \mid 0,\lambda_1-\lambda_2\rangle \times \hat{\mathbf{T}}^{\mathbf{1}^{+-}}_{\mathbf{1,0,0,1}} +$$
$$+\sqrt{2/3} \times \langle 1,0,1,\lambda_4 \mid 1,\lambda_4\rangle \times \sqrt{1/3}\langle 0,0,1,\lambda_1-\lambda_2 \mid 1,\lambda_1-\lambda_2\rangle \times$$
$$\times \langle 1/2,\lambda_1,1/2,-\lambda_2 \mid 1,\lambda_1-\lambda_2\rangle \times \hat{\mathbf{T}}^{\mathbf{1}^{--}}_{\mathbf{0,1,1,1}} +$$
$$+\sqrt{2/3}\langle 1,0,1,\lambda_4 \mid 1,\lambda_4\rangle \times \sqrt{5/3}\langle 2,0,1,\lambda_1-\lambda_2 \mid 1,\lambda_1-\lambda_2\rangle \times$$
$$\times \langle 1/2,\lambda_1,1/2,-\lambda_2 \mid 1,\lambda_1-\lambda_2\rangle \times \hat{\mathbf{T}}^{\mathbf{1}^{--}}_{\mathbf{2,1,1,1}} +$$
$$+\sqrt{5/3}\langle 2,0,1,\lambda_4 \mid 1,\lambda_4\rangle \times [\sqrt{3/3}\,\langle 1,0,0,\lambda_1-\lambda_2 \mid 1,\lambda_1-\lambda_2\rangle \times$$
$$\times \langle 1/2,\lambda_1,1/2,-\lambda_2 \mid 0,\lambda_1-\lambda_2\rangle \times \hat{\mathbf{T}}^{\mathbf{1}^{+-}}_{\mathbf{1,0,2,1}}..........] \times \lambda_6 \Big|^2 \tag{22}$$

That means:When you compare runs at different $\overline{p}$-momenta,you have to normalize by the different N_{init} values and by $\frac{1}{p_{\overline{p}}^2}$.

In the following the decomposition into pairs of λ_1 and λ_2 is performed.λ_6 is set to +1.The term with λ_6=-1 is similar to the one discussed here and exhibits the same amplitudes.

$$
\begin{aligned}
w = |\sum_J (2J+1) \sum_{\lambda_4} d^J_{0,\lambda_4}(\Theta_{34})\, D^{1*}_{\lambda_4 \lambda_5 - \lambda_6}(\Omega_{56}\prime) \sum_{L_{12},S_{12}} \sqrt{\frac{2L_{12}+1}{2J+1}} \langle L_{12},0,S_{12},0 \mid J,0\rangle \times \\
\times \langle 1/2,1/2,1/2,-1/2 \mid S,0\rangle \sum_{L_{34},S_{34}(=1)} \sqrt{\frac{2L_{34}+1}{2J+1}} \langle L_{34},0,1,\lambda_4 \mid J,\lambda_4\rangle \times \\
\times \hat{T}^{J^{PC}}_{L_{12},S_{12},L_{34},S_{34}=1}\Big|^2 + \qquad (\lambda_1 = 1/2; \lambda_2 = 1/2; M = \lambda_1 - \lambda_2 = 0) \\
+|\sum_J (2J+1) \sum_{\lambda_4} d^J_{0,\lambda_4}(\Theta_{34})\, D^{1*}_{\lambda_4 \lambda_5 - \lambda_6}(\Omega_{56}\prime) \times \sum_{L_{12},S_{12}} \sqrt{\frac{2L_{12}+1}{2J+1}} \langle L_{12},0,S_{12},0 \mid J,0\rangle \times \\
\langle 1/2,-1/2,1/2,1/2 \mid S,0\rangle \sum_{L_{34},S_{34}(=1)} \sqrt{\frac{2L_{34}+1}{2J+1}} \langle L_{34},0,1,\lambda_4 \mid J,\lambda_4\rangle \times \\
\times \hat{T}^{J^{PC}}_{L_{12},S_{12},L_{34},S_{34}=1}\Big|^2 + \qquad (\lambda_1 = -1/2; \lambda_2 = -1/2; M = 0) \\
+|\sum_J (2J+1) \sum_{\lambda_4} d^J_{1,\lambda_4}(\Theta_{34})\, D^{1*}_{\lambda_4 \lambda_5 - \lambda_6}(\Omega_{56}\prime) \times \sum_{L_{12},S_{12}} \sqrt{\frac{2L_{12}+1}{2J+1}} \langle L_{12},0,S_{12},1 \mid J,1\rangle \times \\
\times \langle 1/2,1/2,1/2,1/2 \mid S_{12},1\rangle \times \sum_{L_{34},S_{34}(=1)} \sqrt{\frac{2L_{34}+1}{2J+1}} \langle L_{34},0,1,\lambda_4 \mid J,\lambda_4\rangle \times \\
\times \hat{T}^{J^{PC}}_{L_{12},S_{12},L_{34},S_{34}=1}\Big|^2 + \qquad (\lambda_1 = 1/2; \lambda_2 = -1/2; M = 1) \\
+|\sum_J (2J+1) \sum_{\lambda_4} d^J_{-1,\lambda_4}(\Theta_{34})\, D^{1*}_{\lambda_4 \lambda_5 - \lambda_6}(\Omega_{56}\prime) \times \sum_{L_{12},S_{12}} \sqrt{\frac{2L_{12}+1}{2J+1}} \langle L_{12},0,S_{12},-1 \mid J,-1\rangle \times \\
\times \langle 1/2,-1/2,1/2,-1/2 \mid S_{12},-1\rangle \times \sum_{L_{34},S_{34}(=1)} \sqrt{\frac{2l+1}{2J+1}} \langle L_{34},0,1,\lambda_4 \mid J,\lambda_4\rangle \times \\
\times \hat{T}^{J^{PC}}_{L_{12},S_{12},L_{34},S_{34}=1}\Big|^2 \qquad (\lambda_1 = -1/2; \lambda_2 = 1/2; M = -1)
\end{aligned} \tag{23}
$$

The two terms with M=0 contain singlett(S_{12}=0) and triplett(S_{12}=1) contributions.The terms with M=1 and M=-1 contain only triplett contributions.The two M=0-terms are ordered according to these contributions:

$$
\begin{aligned}
w \;=\; & \Big|\sum_J [\cdots \sum_{L_{12},S_{12}(=0)} \cdots \langle L_{12},0,0,0 \mid J,0\rangle \times \underbrace{\langle 1/2,1/2,1/2,-1/2 \mid 0,0\rangle}_{\sqrt{1/2}} \times \\
& \times \sum_{L_{34},S_{34}(=1)} \cdots \hat{T}^{J^{PC}}_{L_{12},0,L_{34},S_{34}(=1)}(singlett) \;+ \\
& + \cdots \sum_{L_{12},S_{12}(=1),} \cdots \langle L_{12},0,1,0 \mid J,0\rangle \times \underbrace{\langle 1/2,1/2,1/2,-1/2 \mid 1,0\rangle}_{\sqrt{1/2}} \times \\
& \times \sum_{L_{34},S_{34}(=1)} \cdots \hat{T}^{J^{PC}}_{L_{12},1,L_{34},S_{34}(=1)}(triplett)]\Big|^2 \;+ \\
& + \Big|\sum_J [\cdots \sum_{L_{12},S_{12}(=0)} \cdots \langle L_{12},0,0,0 \mid J,0\rangle \times \underbrace{\langle 1/2,-1/2,1/2,1/2 \mid 0,0\rangle}_{-\sqrt{1/2}} \times \\
& \times \sum_{L_{34},S_{34}(=1)} \cdots \hat{T}^{J^{PC}}_{L_{12},0,L_{34},S_{34}(=1)}(singlett) \;+ \\
& + \cdots \sum_{L_{12},S_{12}(=1),} \cdots \langle L_{12},0,1,0 \mid J,0\rangle \times \underbrace{\langle 1/2,1/2,1/2,-1/2 \mid 1,0\rangle}_{\sqrt{1/2}} \times \\
& \times \sum_{L_{34},S_{34}(=1)} \cdots \hat{T}^{J^{PC}}_{L_{12},1,L_{34},S_{34}(=1)}(triplett)]\Big|^2 \;+ \\
& + \Big| \cdots \text{as above} \cdots \Big|^2 + \qquad (M=1) \\
& + \Big| \cdots \text{as above} \cdots \Big|^2 + \qquad (M=-1)
\end{aligned}
\tag{24}
$$

The M=0 terms have the following structure(A,B complex numbers):

$$
\big|A \,+\, B\big|^2 \,+\, \big|-A \,+\, B\big|^2 = 2 \times \big|A\big|^2 \,+\, 2 \times \big|B\big|^2 \tag{25}
$$

The final amplitude is(only the term with $\lambda_6 = 1$ is given explicitly;the term with $\lambda_6 = -1$ exhibits the same amplitudes)

$$\frac{d^3\sigma}{d\cos\Theta_{34} d\cos\Theta_{56} d\Phi_{56'}} = 2\pi \times a_{iso}^2 \times \frac{3\, p_\gamma}{512\,\pi^3\, p_{\bar{p}}^2 m_\omega^2\, \Gamma_\omega} \times w \tag{26}$$

$$\begin{aligned}
w \;=\; & 2 \times \Big| \sum_J (2J+1) \sum_{\lambda_4} d^J_{0,\lambda_4}(\Theta_{34})\, D^{1*}_{\lambda_4 \lambda_5 - \lambda_6}(\Omega_{56'}) \sum_{L_{12}, S_{12}=0,} \sqrt{\frac{2L_{12}+1}{2J+1}} \langle L_{12}, 0, 0, 0 \mid J, 0\rangle \times \\
& \times \underbrace{\langle 1/2, 1/2, 1/2, -1/2 \mid 0, 0\rangle}_{\sqrt{1/2}} \sum_{L_{34}, S_{34}(=1)} \sqrt{\frac{2L_{34}+1}{2J+1}} \langle L_{34}, 0, 1, \lambda_4 \mid J, \lambda_4\rangle \times \\
& \times \hat{T}^{J^{PC}}_{L_{12},0,L_{34},S_{34}=1}(singlett)\Big|^2 + \qquad (M=0, singlett) \\
& +2 \times \Big| \sum_J (2J+1) \sum_{\lambda_4} d^J_{0,\lambda_4}(\Theta_{34})\, D^{1*}_{\lambda_4 \lambda_5 - \lambda_6}(\Omega_{56'}) \times \sum_{L_{12}, S_{12}=1,} \sqrt{\frac{2L_{12}+1}{2J+1}} \langle L_{12}, 0, 1, 0 \mid J, 0\rangle \times \\
& \underbrace{\langle 1/2, -1/2, 1/2, 1/2 \mid 1, 0\rangle}_{\sqrt{1/2}} \sum_{L_{34}, S_{34}(=1)} \sqrt{\frac{2L_{34}+1}{2J+1}} \langle L_{34}, 0, 1, \lambda_4 \mid J, \lambda_4\rangle \times \\
& \times \hat{T}^{J^{PC}}_{L_{12},1,L_{34},S_{34}=1}(triplett)\Big|^2 + \qquad (M=0, triplett) \\
& + \Big| \sum_J (2J+1) \sum_{\lambda_4} d^J_{1,\lambda_4}(\Theta_{34})\, D^{1*}_{\lambda_4 \lambda_5 - \lambda_6}(\Omega_{56'}) \times \sum_{L_{12}, S_{12}=1,} \sqrt{\frac{2L_{12}+1}{2J+1}} \langle L_{12}, 0, 1, 1 \mid J, 1\rangle \times \\
& \times \underbrace{\langle 1/2, 1/2, 1/2, 1/2 \mid 1, 1\rangle}_{1} \times \sum_{L_{34}, S_{34}(=1)} \sqrt{\frac{2L_{34}+1}{2J+1}} \langle L_{34}, 0, 1, \lambda_4 \mid J, \lambda_4\rangle \times \\
& \times \hat{T}^{J^{PC}}_{L_{12},1,L_{34},S_{34}=1}(triplett)\Big|^2 + \qquad (M=1, triplett) \\
& + \Big| \sum_J (2J+1) \sum_{\lambda_4} d^J_{-1,\lambda_4}(\Theta_{34})\, D^{1*}_{\lambda_4 \lambda_5 - \lambda_6}(\Omega_{56'}) \times \sum_{L_{12}, 1} \sqrt{\frac{2L_{12}+1}{2J+1}} \langle L_{12}, 0, 1, -1 \mid J, -1\rangle \times \\
& \times \underbrace{\langle 1/2, -1/2, 1/2, -1/2 \mid 1, -1\rangle}_{1} \times \sum_{L_{34}, S_{34}(=1)} \sqrt{\frac{2L_{34}+1}{2J+1}} \langle L_{34}, 0, 1, \lambda_4 \mid J, \lambda_4\rangle \times \\
& \times \hat{T}^{J^{PC}}_{L_{12},1,L_{34},S_{34}=1}(triplett)\Big|^2 \qquad (M=-1, triplett)
\end{aligned} \tag{27}$$

Note:Only terms withL_{12}, S_{12}, L_{34} andS_{34} compatible with the conservation of J,P and C are $\neq 0$.
Explicit examples for J=1 and J=2 are given in Appendix C.

2.7 Comparison with the Standard Method

The Standard Method was used in various analyses of $\overline{p}p$ reactions in flight and is described in detail in [10],[11] and [9],e.g.It is based on the decays of states with a definite J^{PC} value,but the coupling to the $\overline{p}p$ helicity states is only taken crudely into account.
The contributing J^{PC} states up to J=6 are given in Table 2 (from [10]).

J	Singlett $\lambda = 0$	J^{PC}	Triplett $\lambda = \pm 1$	J^{PC}	Triplett $\lambda = 0, \pm 1$	J^{PC}
0	$^1S_0$	0^{-+}			$^3P_0$	0^{++}
1	$^1P_1$	1^{+-}	$^3P_1$	1^{++}	$^3S_1,^3D_1$	1^{--}
2	$^1D_2$	2^{-+}	$^3D_2$	2^{--}	$^3P_2,^3F_2$	2^{++}
3	$^1F_3$	3^{+-}	$^3F_3$	3^{++}	$^3D_3,^3G_3$	3^{--}
4	$^1G_4$	4^{-+}	$^3G_4$	4^{--}	$^3F_4,^3H_4$	4^{++}
5	$^1H_5$	5^{+-}	$^3H_5$	5^{++}	$^3G_5,^3I_5$	5^{--}
6	$^1I_6$	6^{-+}	$^3I_6$	6^{--}	$^3H_6,^3J_6$	6^{++}

Table 2: Fermion-Antifermion initial states

The weight factor is given by [10]

$$w = 2\left|Singl., M=0\right|^2 + 2\left|Tripl., M=0\right|^2 + \left|Tripl., M=1\right|^2 + \left|Tripl., M=-1\right|^2$$

The helicity description reproduces the contributing initial J^{PC} states.Non contributing terms are zero either because of C- or P-violation or zero C.G. coefficients due to the coupling of the J^{PC} states to the initial $\overline{p}p$ states.
Also the structure of the weight factor is reproduced, as well as the number of fit parameters per given J-value.
The amplitude of the Standard Method for the $\overline{p}p \to \omega + \pi^0; \omega \to \pi^0 + \gamma$ reaction for a given J,M and λ_6 is given by [9]

$$A_J^{M,\lambda_6} = \sum_{\lambda_4} (-\sqrt{\pi}) \times d^J_{M,\lambda_4}(\Theta_{34}) D^{1*}_{\lambda_4,\lambda_6}(\Omega_{56}\prime) \sum_{L_{34},S_{34}(=1)} \langle L_{34}, 0, 1, \lambda_4 \mid J, \lambda_4 \rangle \times \lambda_6 \times \alpha^{J^{PC},M}_{L_{34},S_{34}} \tag{28}$$

$\alpha^{J^{PC}}_{L_{34},S_{34}}$ is the product of the J^{PC} production- and decay- amplitudes.

The corresponding amplitude in the helicity description for a given J and for a given (λ_1, λ_2) combination (fixed M) is-apart from kinematical factors-(see (21)):

$$A_J^{M,\lambda_6} = (2J+1)\sum_{\lambda_4} d^J_{M,\lambda_4}(\Theta_{34})\, D^{1*}_{\lambda_4\lambda_5-\lambda_6}(\Omega_{56}\prime) \times \sum_{L_{12},S_{12}} \sqrt{\frac{2L_{12}+1}{2J+1}} \langle L_{12},0,S_{12},M \mid J,M\rangle \times$$

$$\times \langle 1/2,\lambda_1,1/2,-\lambda_2 \mid S_{12},M\rangle \sum_{L_{34},S_{34}(=1)} \sqrt{\frac{2L_{34}+1}{2J+1}} \langle L_{34},0,1,\lambda_4 \mid J,\lambda_4\rangle \times \lambda_6 \times \hat{T}^{J^{PC}}_{L_{12},S_{12},L_{34},S_{34}(=1)} \tag{29}$$

The comparison of both expressions for a given $\lambda_1, \lambda_2(M)$ combination and for given L_{34}, S_{34} values yields the correspondance of the amplitudes of the Standard Method and of the helicity description:

$$(-\sqrt{\pi}) \times \alpha^{J^{PC},M}_{L_{34},S_{34}(=1)} = (2J+1) \times \sqrt{\frac{2L_{34}+1}{2J+1}} \sum_{L_{12},S_{12}} \sqrt{\frac{2L_{12}+1}{2J+1}} \langle L_{12},0,S_{12},M \mid J,M\rangle \times$$

$$\times \langle 1/2,\lambda_1,1/2,-\lambda_2 \mid S_{12},M\rangle \times \hat{T}^{J^{PC}}_{L_{12},S_{12},L_{34},S_{34}(=1)} \tag{30}$$

For M=0,there exist two combinations(Singlett,S_{12}=0;Triplett,S_{12}=1),for M$\pm$ 1 there is only a triplett state.

Examples are given for J=1 and J=2:
J=1:

$$(-\sqrt{\pi}) \times \alpha^{1^{+-},0}_{L_{34},S_{34}(=1)}(singlett) = 3 \times \sqrt{\frac{2L_{34}+1}{3}} \sqrt{1/2} \times \hat{T}^{1^{+-}}_{1,0,L_{34},S_{34}(=1)}$$

$$(-\sqrt{\pi}) \times \alpha^{1^{--},0}_{L_{34},S_{34}(=1)}(triplett) = 3 \times \sqrt{\frac{2L_{34}+1}{3}} [\sqrt{1/6} \times \hat{T}^{1^{--}}_{0,1,L_{34},S_{34}(=1)} - \sqrt{1/3} \times \hat{T}^{1^{--}}_{2,1,L_{34},S_{34}(=1)}]$$

$$(-\sqrt{\pi}) \times \alpha^{1^{--},1}_{L_{34},S_{34}(=1)}(triplett) = 3 \times \sqrt{\frac{2L_{34}+1}{3}} [\sqrt{1/3} \times \hat{T}^{1^{--}}_{0,1,L_{34},S_{34}(=1)} + \sqrt{1/6} \times \hat{T}^{1^{--}}_{2,1,L_{34},S_{34}(=1)}]$$

$$(-\sqrt{\pi}) \times \alpha^{1^{--},-1}_{L_{34},S_{34}(=1)}(triplett) = 3 \times \sqrt{\frac{2L_{34}+1}{3}} [\sqrt{1/3} \times \hat{T}^{1^{--}}_{0,1,L_{34},S_{34}(=1)} + \sqrt{1/6} \times \hat{T}^{1^{--}}_{2,1,L_{34},S_{34}(=1)}] \tag{31}$$

J=2:

$$(-\sqrt{\pi}) \times \alpha^{2^{--},1}_{L_{34},S_{34}(=1)}(triplett) = 5 \times \sqrt{\frac{2L_{34}+1}{5}} (-\sqrt{1/2}) \times \hat{T}^{2^{--}}_{2,1,L_{34},S_{34}(=1)}$$

$$(-\sqrt{\pi}) \times \alpha^{2^{--},-1}_{L_{34},S_{34}(=1)}(triplett) = 5 \times \sqrt{\frac{2L_{34}+1}{5}} \sqrt{1/2} \times \hat{T}^{2^{--}}_{2,1,L_{34},S_{34}(=1)} \tag{32}$$

Note:Negative sign! for J=2,M=1

These examples show,that the use of Standard Method amplitudes and of helicity amplitudes is not equivalent.The Standard Method amplitudes are linear combinations of the helicity amplitudes,which might be still tolerable.The real difference is in their signs,which causes a problem,when coherent sums over J are constructed.E.g.,the Standard Method amplitudes give wrong ω density matrix elements,which don't fulfill the general symmetry rules(see next chapter).
In addition,the helicity amplitudes are well defined,in contrast to the Standard-Method amplitudes,which are an undefined mixture of production and decay amplitudes and of kinematical factors.Only with helicity amplitudes a proper comparison between reactions at different energies is possible.Therefore,future analyses should be performed with helicity amplitudes using the expression(27).
The formulae here are given for $\overline{p}p \to \omega\pi^0$,but can be easily extended to particles with different spins,e.g.$f_0, f_2, ..$,which are discussed in [10] and [11].In these cases,for each resonance a separate amplitude has to be constructed,which are then added coherently.All final state particles are added incoherently,as was here the case with the γ.

3 Spin Density Matrix Formulation

In the following the spin-density-matrix(ρ) formalism for the reaction under study is introduced.The fit of ρ-matrix elements and decay amplitudes from the measured differential cross section(angular distribution) is equivalent to the fit of the helicity amplitudes discussed above.However,sometimes the ρ-matrix elements are calculated from the fitted helicity amplitudes of a full PWA [13, 15].On the other hand,several of the ρ-matrix elements can be determined solely from the measured angular distributions of the decaying particle [12, 17].Furthermore,the spherical moments of the measured angular distribution can be determined using a projection technique and the rho-matrix elements can be calculated.All methods are discussed in the following for the ω-decay.

3.1 Rho-Matrix Formalism

In (2) $|T_{fi}|^2$ can be rewritten in the following form(isospin factor neglected):

$$|T_{fi}|^2 = \sum_{\lambda_1,\lambda_2,\lambda_3,\lambda_5,\lambda_6} \times \sum_{\lambda_4,\lambda_{4'}} [T^*_{\lambda_1,\lambda_2,\lambda_3,\lambda_4} \times T_{\lambda_1,\lambda_2,\lambda_3,\lambda_{4'}} \times A^*_{\lambda_4,\lambda_5-\lambda_6} \times A_{\lambda_{4'},\lambda_5-\lambda_6}] =$$

$$= \{ \sum_{\lambda_1,\lambda_2,\lambda_3,\lambda_4} |T_{\lambda_1\lambda_2\lambda_3\lambda_4}|^2 \} \times \sum_{\lambda_5,\lambda_6} \times \sum_{\lambda_4,\lambda_{4'}} [A^*_{\lambda_4,\lambda_5-\lambda_6} \times \rho_{\lambda_4\lambda_{4'}} \times A_{\lambda_{4'},\lambda_5-\lambda_6}] =$$

$$= \{ \sum_{\lambda_1,\lambda_2,\lambda_3,\lambda_4} \left|T_{\lambda_1\lambda_2\lambda_3\lambda_4}\right|^2\} \times \frac{3}{4\,\pi} \times \sum_{\lambda_5,\lambda_6} \sum_{\lambda_4,\lambda_4\prime} [D^1_{\lambda_4,\lambda_5-\lambda_6}(\Omega_{56}) \times \rho_{\lambda_4\lambda_4\prime} \times D^{1*}_{\lambda_4\prime,\lambda_5-\lambda_6}(\Omega_{56}) \times \left|A^1_{\lambda_5\lambda_6}\right|^2] \tag{33}$$

with the spin density matrix elements for particle 4 (ω)

$$\rho_{\lambda_4\lambda_4\prime} = \frac{1}{\sum_{\lambda_1,\lambda_2,\lambda_3,\lambda_4} \left|T_{\lambda_1\lambda_2\lambda_3\lambda_4}\right|^2} \times \sum_{\lambda_1,\lambda_2,\lambda_3} T^*_{\lambda_1,\lambda_2,\lambda_3,\lambda_4} \times T_{\lambda_1,\lambda_2,\lambda_3,\lambda_4\prime} \tag{34}$$

The ρ-matrix is hermitean and has trace=1 by definition.The diagonal elements are real.It has additional symmetries,if e.g. parity is conserved in the production process.
In our case(Spin=1;Parity Conservation;Quantization axis(z) in the production plane) the ρ-matrix has the form (see [12])

$$\rho^0_{\lambda_4\lambda_4\prime} = \begin{pmatrix} 1/2(1-\rho^0_{00}) & \Re\rho^0_{10} + \imath\Im\rho^0_{10} & \Re\rho^0_{1-1} \\ \Re\rho^0_{10} - \imath\Im\rho^0_{10} & \rho^0_{00} & -(\Re\rho^0_{10} - \imath\Im\rho^0_{10}) \\ \Re\rho^0_{1-1} & -(\Re\rho^0_{10} + \imath\Im\rho^0_{10}) & 1/2(1-\rho^0_{00}) \end{pmatrix} \tag{35}$$

with the four independent parameters ρ^0_{00} , $\Re\rho^0_{10}$, $\Im\rho^0_{10}$ and $\Re\rho^0_{1-1}$.^{0} refers to measurements with unpolarized particles in the initial state(see [12]).

Note:
The matrix elements and their symmetry properties depend critically on the choice of the quantization axis.For an axis perpendicular to the production plane,e.g.,the rho matrix is [18]

$$\rho^0_{\lambda_4\lambda_4\prime} = \begin{pmatrix} \rho^0_{11} & 0 & \rho^0_{1-1} \\ 0 & \rho^0_{00} & 0 \\ \rho^{*0}_{1-1} & 0 & (1-\rho^0_{11}-\rho^0_{00}) \end{pmatrix} \tag{36}$$

No polarization and no alignment for the S=1 case mean:$\rho^0_{11} = \rho^0_{-1-1} = \rho^0_{00} = 1/3$
Alignment means:$\rho^0_{11} = \rho^0_{-1-1} \neq \rho^0_{00}$
Polarization means:$\rho^0_{11} \neq \rho^0_{-1-1}$

The ρ matrix elements are generally dependent on s,$\cos\Theta_{34}(\Phi_{34})$ and can be derived -together with the helicity amplitudes $A^1_{\lambda_5\lambda_6}$-from the measured data using the following expression based on (1),(2) and (33):

$$\frac{d^3N}{d\cos\Theta_{34}d\cos\Theta_{56}d\Phi_{56}\prime} = L_{int} \times 1/4 \times a^2_{iso} \times \frac{1}{4s^{1/2}p_{\overline{p}}} \times \frac{1}{2m_\omega\Gamma_\omega} \times Kin_{34} \times Kin_{56} \times$$

$$\times \{ \sum_{\lambda_1,\lambda_2,\lambda_3,\lambda_4} \left|T_{\lambda_1\lambda_2\lambda_3\lambda_4}\right|^2\} \times \frac{3}{4\pi} \sum_{\lambda_5,\lambda_6} \sum_{\lambda_4,\lambda_4\prime} [D^1_{\lambda_4,\lambda_5-\lambda_6}(\Omega_{56}\prime) \times \rho_{\lambda_4\lambda_4\prime} \times D^{1*}_{\lambda_4\prime,\lambda_5-\lambda_6}(\Omega_{56}\prime) \times \left|A^1_{\lambda_5\lambda_6}\right|^2] =$$

$$= L_{int}\times\frac{1}{2m_\omega\Gamma_\omega}\times Kin_{56}\times\frac{3}{4\pi}\times\frac{d\sigma_{prod}}{d\cos\Theta_{34}}\sum_{\lambda_5,\lambda_6}\sum_{\lambda_4,\lambda_{4'}}[D^1_{\lambda_4,\lambda_5-\lambda_6}(\Omega_{56'})\times\rho_{\lambda_4\lambda_{4'}}\times D^{1*}_{\lambda_{4'},\lambda_5-\lambda_6}(\Omega_{56'})\times\left|A^1_{\lambda_5\lambda_6}\right|^2] \quad (37)$$

with
$\rho_{\lambda_4\lambda_{4'}} = \rho_{\lambda_4\lambda_{4'}}(s,\cos\Theta_{34})$ and

$$\frac{d\sigma_{prod}}{d\cos\Theta_{34}} = 1/4\times a^2_{iso}\times\frac{1}{4s^{1/2}p_{\overline{p}}}\times Kin_{34}\times\sum_{\lambda_1,\lambda_2,\lambda_3,\lambda_4}\left|T_{\lambda_1\lambda_2\lambda_3\lambda_4}\right|^2 \quad (38)$$

(37) is equivalent to(21)and can be used for the analysis of the complete reaction chain.Here the fit parameters(for fixed s) are no more the T-amplitudes,but the rho-matrix elements for binned $\cos\Theta_{34}$ values,together with the amplitudes A^1_{56}.The number of fit parameters is not the same as in (21),but in the same ball park.The number will also increase with s.
In many cases the rho-matrix elements are not fitted from(37),but are determined using the $T_{\lambda_1,\lambda_2,\lambda_3,\lambda_4}$-amplitudes taken from a full PWA-analysis using (34).This procedure was used in [13, 14, 15].Here,the production amplitudes for specific λ_4 values must be known.This is only possible by using the PWA parameters from an analysis of production and decay(see(21)).A projection of the rho-matrix elements from the measured cross section(angular distribution) is not feasible,in contrast to the determination of spherical moments discussed later.
Not all elements of the rho-matrix can be determined from the measured 3-dimensional event distribution.Here,$\Im\rho_{10}$ is not determinable(see also (42)).As will be shown later it is proportional to P_y,the omega vector polarization perpendicular to the production plane.It can only be determined using polarized initial states.
Note:
In electromagnetic and electroweak interactions the rho-matrix elements can be exactly calculated(see, e.g, [16],pages 372 ff.)

3.2 Rho Matrix Determination from decay angular distributions

Several elements of ρ -for a specific production angle or averaged over the whole production angle-can also be derived from the measurement of the angular distribution of the decaying particle $4(\omega)$ only (Schilling's Method) (see [12, 13, 14, 15])

In our case for a selected production angle bin($\Delta\Theta_{34}$) the 2-dimensional event distribution is given by (see(37))

$$\frac{dN_{\Delta\Theta_{34}}}{d\cos\Theta_{56}d\Phi_{56'}} = \int_{\Theta_{34}}^{\Theta_{34}+\Delta\Theta_{34}}\frac{dN}{d\cos\Theta_{34}d\cos\Theta_{56}d\Phi_{56'}}d\cos\Theta_{34} =$$

$$= L_{int}\times\frac{1}{2m_\omega\Gamma_\omega}\times Kin_{56}\times\frac{3}{4\pi}\times\sum_{\lambda_4,\lambda_{4'}}\times\sum_{\lambda_5,\lambda_6}[D^{1*}_{\lambda_4,\lambda_5-\lambda_6}(\Omega_{56'})\times$$

$$\times \underbrace{\int_{\Theta_{34}}^{\Theta_{34}+\Delta\Theta_{34}} \frac{d\sigma_{prod}}{d\cos\Theta_{34}} \times \rho^0_{\lambda_4\lambda_4\prime} d\cos\Theta_{34}}_{\rho^0_{\lambda_4\lambda_4\prime}(\Delta\Theta_{34})\times\sigma_{\Delta\Theta_{34}}} \times D^1_{\lambda_4\prime,\lambda_5-\lambda_6}(\Omega_{56}\prime) \times \left|A^1_{\lambda_5\lambda_6}\right|^2] \quad (39)$$

with the binned ρ-matrix elements $\rho^0_{\lambda_4\lambda_4\prime}(\Delta\Theta_{34})$ and the binned cross section $\sigma_{\Delta\Theta_{34}}$ From now on the index $\Delta\Theta_{34}$ will be omitted.dN,N,..always are understood as given for the specific production bin.It can be extended to the full range of the production angle.In that case the average over all production angles is given.

The angular distribution is given as

$$I(\Omega_{56}\prime) = \frac{dN/N}{d\cos\Theta_{56} d\Phi_{56}\prime} =$$
$$= \frac{3}{4\pi} \times \sum_{\lambda_4,\lambda_4\prime} \times \sum_{\lambda_5,\lambda_6} [D^{1*}_{\lambda_4,\lambda_5-\lambda_6}(\Omega_{56}\prime) \times \rho_{\lambda_4\lambda_4\prime} D^1_{\lambda_4\prime,\lambda_5-\lambda_6}(\Omega_{56}\prime) \times \left|A^1_{\lambda_5\lambda_6}\right|^2 \{\sum_{\lambda_5\lambda_6} \left|A^1_{\lambda_5\lambda_6}\right|^2\}^{-1}] \quad (40)$$

with

$$N = L_{int} \times \frac{1}{2m_\omega\Gamma_\omega} \times Kin_{56} \times \sigma \times \sum_{\lambda_5\lambda_6} \left|A^1_{\lambda_5\lambda_6}\right|^2 \quad (41)$$

The expression(40) results to:

$$I(\Omega_{56}\prime) \propto [1/2(1+\rho^0_{00}) + 1/2(1-3\rho^0_{00})\cos\Theta_{56}{}^2 + \\ + \rho^0_{1-1}\sin\Theta_{56}{}^2 \cos 2\phi_{56}\prime + \sqrt{2}\,\Re\rho^0_{10} \sin 2\Theta_{56} \cos\phi_{56}\prime] \quad (42)$$

Note,that only three of the four independent ρ matrix elements can be determined by this method.Only when the initial state is polarized,the angle Φ_{34} is defined and further elements of ρ are measurable.

The expression for $I(\cos\Theta_{56})$ was worked out from $\int I(\Omega_{56\prime}) d\Phi_{56}\prime$ (42) yielding

$$I(\cos\Theta_{56}) = \frac{dN}{d\cos\Theta_{56}} \propto [(1+\rho^0_{00}) + (1-3\rho^0_{00}) \times \cos\Theta_{56}{}^2] \quad (43)$$

Here,only diagonal elements of ρ^0 contribute and can be determined.
The elements of ρ^0 determined from (21,34) and from the fit of angular distributions(42,43) must be identical.This has been demonstrated in [13, 14, 15].

In cases,where not the full reaction chain can be analyzed(too many parameters or incomplete measurements), equations like (42,43)are useful in determining the J^P values of unknown resonances,analyzing their decay distributions.That is particularly efficient,when not only one decay but a decay chain is available [17].That is further discussed in a parallel note [25].

3.3 Multipole Expansion of Rho-Matrix Elements

The Rho-matrix elements for a resonance of spin S as defined in (34) can be expanded in a system of complete functions Q_{LM}

$$\rho^S_{\lambda_4\lambda_4\prime} = \frac{1}{2S+1}\sum_{L=0}^{2S}\sum_{M=-L}^{L} q^*_{LM}(Q^S_{LM})_{\lambda_4\lambda_4\prime} \tag{44}$$

with the spherical momentum tensors,restricted to certain values(see [26])

$$q^*_{LM} = \langle Q^\dagger_{LM}\rangle = Tr(\rho\, Q^\dagger_{ML}) \tag{45}$$

normalized to

$$Tr(Q_{LM}\, Q^\dagger_{L\prime M\prime}) = (2S+1)\delta_{LL\prime\delta_{MM\prime}} \tag{46}$$

This normalization is used,e.g.,in [19, 23, 22, 24]
With the normalization

$$Tr(Q_{LM}\, Q^\dagger_{L\prime M\prime}) = \frac{(2S+1)}{(2L+1)}\delta_{LL\prime\delta_{MM\prime}} \tag{47}$$

the expression for Rho is

$$\rho^S_{\lambda_4\lambda_4\prime} = \frac{1}{2S+1}\sum_{L=0}^{2S}\sum_{M=-L}^{L} (2L+1)\, q^*_{LM}(Q^S_{LM})_{\lambda_4\lambda_4\prime} \tag{48}$$

This normalization is used,e.g.,in [4, 20, 6]
(Occasionally also $Tr(Q_{LM}\, Q^\dagger_{L\prime M\prime}) = \delta_{LL\prime\delta_{MM\prime}}$ is used [18])
All operators(functions,matrices)fullfilling (46) or (47) can be used for the expansion.
Very convenient operators Q_{LM} for a resonance with spin S are Q^S_{LM} with the matrix elements

$$\langle\lambda_4\prime|Q^S_{LM}|\lambda_4\rangle = (Q^S_{LM})_{\lambda_4\lambda_4\prime} = \alpha\times(-1)^{S-\lambda_4\prime}\langle S\lambda_4 S-\lambda_4\prime|L\,M\rangle \tag{49}$$

This expression can be considered as a matrix with the indices $\lambda_4, \lambda_4\prime$.For S=1 it has nine components:

$$(Q^S_{LM}) = \alpha\times\begin{pmatrix} \langle 1,1,1,-1|L,M\rangle & -\langle 1,1,1,0|L,M\rangle & \langle 1,1,1,1|L,M\rangle \\ \langle 1,0,1,-1|L,M\rangle & -\langle 1,0,1,0|L,M\rangle & \langle 1,0,1,1|L,M\rangle \\ \langle 1,-1,1,-1|L,M\rangle & -\langle 1,-1,1,0|L,M\rangle & \langle 1,-1,1,1|L,M\rangle \end{pmatrix} \tag{50}$$

α is dependent on the normalization of Q_{LM}:
$\alpha = \alpha_s = \sqrt{2S+1}$ for (46) $\qquad \alpha = \alpha_{SL} = \sqrt{\frac{2S+1}{2L+1}}$ for (47)

For the normalization $\alpha_s = \sqrt{(2S+1)}$ the rho-matrix assumes the following form

$$\rho^S_{\lambda_4\lambda_4\prime} = \frac{1}{2S+1}\sum_{L=0}^{2S}\sum_{M=-L}^{L} q^{S*}_{LM} \times \alpha \times (-1)^{S-\lambda_4\prime}\langle S\lambda_4 S-\lambda_4\prime \big| L\, M\rangle \tag{51}$$

The inverse is

$$q^{S*}_{LM} = (2S+1) \times \sum_{\lambda_4\lambda_4\prime} \alpha^{-1}\,(-1)^{S-\lambda_4\prime}\langle S\lambda_4 S-\lambda_4\prime \big| L\, M\rangle\, \rho^S_{\lambda_4\lambda_4\prime} \tag{52}$$

For the normalization $\alpha_{SL} = \sqrt{\frac{2S+1}{2L+1}}$

$$\rho^S_{\lambda_4\lambda_4\prime} = \frac{1}{2S+1}\sum_{L=0}^{2S}\sum_{M=-L}^{L} (2L+1)\, q^{S*}_{LM} \times \alpha \times (-1)^{S-\lambda_4\prime}\langle S\lambda_4 S-\lambda_4\prime \big| L\, M\rangle \tag{53}$$

Examples for Q^S_{LM} for S=1 and either α are given below(underline means operator/matrix).Q^S_{LM} can be expressed in terms of vectors and tensors formed from $\underline{S_x}, \underline{S_y}, \underline{S_z}$:

$$Q^1_{00} = \alpha\,\sqrt{1/3}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \alpha\sqrt{1/3}\,\underline{1}$$

$$Q^1_{10} = \alpha\,\sqrt{1/2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} = \alpha\sqrt{1/2}\,\underline{S_z}$$

$$Q^1_{11} = -\alpha\,1/2\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} = -\alpha\,1/2\,(\underline{S_x} + \imath\underline{S_y})$$

$$Q^1_{1-1} = \alpha\,1/2\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} = \alpha\,1/2\,(\underline{S_x} - \imath\underline{S_y})$$

$$Q_{20} = \alpha\,\sqrt{1/6}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \alpha\,3\,\sqrt{1/6}\,(\underline{S_z^2} - 2/3 \times \underline{1}) \tag{54}$$

The other Q-matrices are not explicitly calculated,but are only given in terms of $\underline{S_x}, \underline{S_y}, \underline{S_z}$:

$$Q^1_{2\pm1} = \mp\alpha\frac{1}{2\sqrt{3}}\,[(\underline{S_x} \pm \imath\underline{S_y})\underline{S_z} + \underline{S_z}(\underline{S_x} \pm \imath\underline{S_y})]$$

$$Q^1_{2\pm2} = \alpha\frac{1}{2\sqrt{3}}\,(\underline{S_x} \pm \imath\underline{S_y})^2 \tag{55}$$

with

$$\underline{S_x} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \qquad \underline{S_y} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -\imath & 0 \\ \imath & 0 & -\imath \\ 0 & \imath & 0 \end{pmatrix} \qquad \underline{S_z} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

(56)

The corresponding expectation values $\langle Q^{s\dagger}_{LM}\rangle = q^*_{LM}$ are:
$\langle Q^{1\dagger}_{00}\rangle = q^{1*}_{00} = \alpha\sqrt{1/3}\,\langle\underline{1}\rangle$
$\langle Q^{1\dagger}_{10}\rangle = q^{1*}_{10} = \alpha\sqrt{1/2}\langle\underline{S_z}^\dagger\rangle = \alpha\sqrt{1/2}\times P_z$ (P_z = vector polarization in z-dir.)
$\langle Q^{1\dagger}_{1\pm1}\rangle = q^{1*}_{1\pm1} = \mp\alpha\times 1/2(\langle\underline{S_x}^\dagger\rangle \mp \langle\underline{S_y}^\dagger\rangle) = \mp\alpha\times 1/2(P_x \mp P_y)$
and so on ...
The rho-matrix for spin 1 can be written (indepedent of the normalization) as

$$\rho^S_{\lambda_4\lambda_4\prime} = \frac{1}{3}\sum_{L=0}^{2S}\sum_{M=-L}^{L} q^*_{LM}(Q^S_{LM}) = 1/3\times(\underline{1} + 3/2\,\underline{\vec{P}}\,\underline{\vec{S}} + 3\,\underline{P_{ij}}\,\underline{S_{ij}}) \qquad (57)$$

with $\vec{P}$= polarization vector and P_{ij} = polarization tensor
and $\vec{S} = (\underline{S_x}, \underline{S_y}, \underline{S_z})$ $\quad \underline{S_{ij}} = 1/2\,(\underline{S_i}\underline{S_j} + \underline{S_j}\underline{S_i}) - 2/3\,\delta_{ij}$ (i,j=x,y,z)

For S=1 and $\alpha_S = \sqrt{3}$ ρ^0_{11} is calculated explicitly as example:

$$\begin{aligned}
\rho^1_{11} &= \frac{1}{3}\sum_{L=0}^{2S}\sum_{M=-L}^{L} q^{S*}_{LM}\times\alpha\times(-1)^{S-\lambda_4\prime}\langle 111-1\big|L\,M\rangle = \\
&= \frac{1}{3}[q^*_{00}\sqrt{3}\,\underbrace{\langle 111-1\big|0\,0\rangle}_{\sqrt{1/3}} + q^*_{10}\sqrt{3}\,\underbrace{\langle 111-1\big|1\,0\rangle}_{\sqrt{1/2}} + \\
&+ q^*_{20}\sqrt{3}\,\underbrace{\langle 111-1\big|2\,0\rangle}_{\sqrt{1/6}}] = \quad \text{(all other LM-terms vanish)} \\
&= 1/3\,[1 + 3/2P_z + \sqrt{1/2}q^*_{20}] \\
&\text{with} \quad q^*_{10} = \alpha_1\sqrt{1/2}\,P_z \quad \text{(in agreement with [23])}
\end{aligned} \qquad (58)$$

⌈ For the normalization $\alpha_{SL} = \sqrt{\frac{2S+1}{2L+1}}$ the ρ^1_{11} matrix element is:
$\rho_{11} = 1/3\,[1 + 3\sqrt{1/2}\,q^*_{10} + \sqrt{5/2}\,q^*_{20}] = 1/3\,[1 + 3/2\,P_z + \sqrt{5/2}\,q^*_{20}]$
with $q^*_{10} = \alpha_{11}\sqrt{1/2}\,P_z$⌋
The complete rho-matrix for spin 1 and $\alpha_S = \sqrt{3}$ is (see [23]):

$$\rho^0_{\lambda_4\lambda_4\prime} = 1/3\times\begin{pmatrix} 1 + 3/2\,P_z + \sqrt{1/2}\,q^*_{20} & 3/2\,P_{1-1} + \sqrt{3/2}\,q^*_{2-1} & \sqrt{3}\,q^*_{2-2} \\ -3/2\,P_{11} - \sqrt{3/2}\,q^*_{21} & 1 - \sqrt{2}\,q^*_{20} & 3/2\,P_{1-1} - \sqrt{3/2}\,q^*_{2-1} \\ \sqrt{3}\,q^*_{22} & -3/2\,P_{11} + \sqrt{3/2}\,q^*_{21} & 1 - 3/2\,P_z + \sqrt{1/2}\,q^*_{20} \end{pmatrix} \qquad (59)$$

with $P_{1\pm1} = \mp\sqrt{1/2}(P_x \pm \imath P_y)$. In case of parity conservation the matrix is further reduced([23]).

From the measured angular distribution $I(\Omega_{56\prime})(40)$ the q-moments for specific values of $L\prime, M\prime$ can be projected out,using the orthogonality of the D-functions:

Using the relations
$D^{j*}_{mm\prime} = D^{j}_{-m-m\prime}(-1)^{m-m\prime}$ and
$D^{j_1}_{m_1m_{1\prime}} D^{j_2}_{m_2m_{2\prime}} = \sum_j \langle j_1m_1j_2m_2 \big| jm\rangle\langle j_1m_{1\prime}j_2m_{2\prime}\big| jm_{\prime}\rangle D^{j}_{mm\prime}$ with
$m = m_1 + m_2$ *and* $m\prime = m_{1\prime} + m_{2\prime}$
the product $D^{1*}_{\lambda_4,-\lambda_6}\, D^{1}_{\lambda_4\prime,-\lambda_6}$ in (40) can be written as

$$D^{1*}_{\lambda_4,-\lambda_6}\, D^{1}_{\lambda_4\prime,-\lambda_6} = D^{1}_{-\lambda_4,\lambda_6}\,(-1)^{\lambda_4+\lambda_6}\, D^{1}_{\lambda_4\prime,-\lambda_6} =$$

$$= \sum_L \underbrace{D^{L}_{-\lambda_4+\lambda_4\prime,0}}_{D^{L*}_{\lambda_4-\lambda_4\prime,0}(-1)^{-\lambda_4+\lambda_4\prime}} \times \underbrace{\langle 1-\lambda_4 1\lambda_4\prime \big| L,-\lambda_4+\lambda_4\prime\rangle}_{\langle 1\lambda_4 1-\lambda_4\prime \big| L,\lambda_4-\lambda_4\prime\rangle\,(-1)^{1+1-L}} \times \underbrace{\langle 1\lambda_6 1-\lambda_6 \big| L,0\rangle}_{\langle 1-\lambda_6 1\lambda_6 \big| L,0\rangle\,(-1)^{1+1-L}} (-1)^{\lambda_4+\lambda_6} =$$

$$= \sum_L D^{L*}_{M,0}\,\langle 1\lambda_4 1-\lambda_4\prime \big| LM\rangle\langle 1-\lambda_6 1\lambda_6\big| L,0\rangle(-1)^{\lambda_4\prime+\lambda_6} \quad \text{with}\, M = \lambda_4-\lambda_4\prime \tag{60}$$

With (51) the angular distribution (40) can be written as:

$$I(\Omega_{56\prime}) = \frac{3}{4\pi}\frac{\alpha}{3}\sum_{\lambda_6}\big|A_{\lambda_6}\big|^2 \sum_L\sum_{L\prime}\sum_{M\prime} q^*_{L\prime M\prime} D^{L*}_{M0}\times$$

$$\times \underbrace{\sum_{\lambda_4\lambda_4\prime}\langle 1\lambda_4 1-\lambda_4\prime\big| LM\rangle\langle 1\lambda_4 1-\lambda_4\prime\big| L\prime M\prime\rangle(-1)^{\lambda_4\prime+\lambda_6}(-1)^{1-\lambda_4\prime}}_{\delta_{LL\prime}\delta_{MM\prime}(-1)^{1+\lambda_6}}\langle 1-\lambda_6 1\lambda_6\big| L\prime 0\rangle\{\sum_{\lambda_6}\big|A^1_{\lambda_6}\big|^2\}^{-1} =$$

$$= \frac{\alpha}{4\pi}\sum_{\lambda_6}\big|A_{\lambda_6}\big|^2 \sum_L q^*_{LM} D^{L*}_{M0}\,\langle 1-\lambda_6 1\lambda_6\big| L0\rangle(-1)^{1+\lambda_6}\times\{\sum_{\lambda_6}\big|A^1_{\lambda_6}\big|^2\}^{-1} \tag{61}$$

The q-moments are projected out from the measured $I(\Omega_{56\prime})$ distribution:
With $\int D^{L*}_{M0}(\Omega_{56\prime})\, D^{L\prime}_{M\prime 0}(\Omega_{56\prime})\, d\Omega_{56\prime} \;=\; \frac{4\pi}{2L\prime+1}\delta_{LL\prime}\delta_{MM\prime}$

$$\int I(\Omega_{56\prime})\, D^{L\prime}_{M\prime 0} d\Omega_{56\prime} \;=$$

$$= \frac{\alpha}{4\pi}\sum_{\lambda_6}\left|A_{\lambda_6}\right|^2 \sum_{L} q^*_{LM}\int D^{L*}_{M0} D^{L\prime}_{M\prime 0} d\Omega_{56\prime}\times$$

$$\times\langle 1-\lambda_6 1\lambda_6\left|L0\right\rangle(-1)^{1+\lambda_6}\times\{\sum_{\lambda_6}\left|A^1_{\lambda_6}\right|^2\}^{-1} \;=$$

$$= \frac{\alpha}{2L\prime+1}\, q^*_{L\prime M\prime}\sum_{\lambda_6}\left|A_{\lambda_6}\right|^2\langle 1-\lambda_6 1\lambda_6\left|L\prime 0\right\rangle(-1)^{1+\lambda_6}\{\sum_{\lambda_6}\left|A^1_{\lambda_6}\right|^2\}^{-1} =$$

$$= \frac{\alpha}{2L\prime+1}\, q^*_{L\prime M\prime}[\left|A_1\right|^2\langle 1-111\left|L\prime 0\right\rangle(-1)^{1+1} \;+\; \left|A_{-1}\right|^2\langle 111-1\left|L\prime 0\right\rangle(-1)^{1-1}]\{\sum_{\lambda_6}\left|A^1_{\lambda_6}\right|^2\}^{-1} =$$

$$= \frac{\alpha}{2L\prime+1}\, q^*_{L\prime M\prime}\{\left|A_1\right|^2+\left|A_{-1}\right|^2\}^{-1}\begin{cases} \sqrt{1/3}\left|A_1\right|^2+\sqrt{1/3}\left|A_{-1}\right|^2 & for L\prime=0\\ -\sqrt{1/2}\left|A_1\right|^2+\sqrt{1/2}\left|A_{-1}\right|^2 & for L\prime=1\\ \sqrt{1/6}\left|A_1\right|^2+\sqrt{1/6}\left|A_{-1}\right|^2 & for L\prime=2\end{cases}$$

$$= \alpha\, q^*_{L\prime M\prime}\begin{cases}\sqrt{1/3} & for L\prime=0\\ 0 & for L\prime=1\\ \frac{1}{5}\sqrt{1/6} & for L\prime=2\end{cases} \tag{62}$$

with $A_1 \;=\; A_{-1}$ (Parity conservation)
The final formula for the determination of q^*_{LM} ($\alpha \;=\; \sqrt{3}$) from the measured angular distribution $I(\Omega_{56\prime})$ is:

$$q^*_{L\prime M\prime} \;=\; \int I(\Omega_{56\prime})\, D^{L\prime}_{M\prime 0} d\Omega_{56\prime}\;\times\begin{cases} 1 & for L\prime=0\\ 0 & for L\prime=1\\ 5\sqrt{2} & for L\prime=2\end{cases} \tag{63}$$

$q^*_{L\prime M\prime}$ vanishes for$L\prime$=1.This is due to parity conservation and means that the vector polarizations do not appear in the expression for rho.

When the moments are determined the rho-matrix elements can be extracted from (51).That is a third method to determine the rho-matrix elements using the measured angular distribution.

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A LS Expansion of the Production Amplitude

Example: J=1

$$\begin{aligned}
&\langle \lambda_4, 0 \mid T^J \mid \lambda_1, \lambda_2 \rangle = \\
&\quad = [\sqrt{1/3}\langle 0,0,1,\lambda_4 \mid 1,\lambda_4\rangle \underbrace{\langle 1,\lambda_4,0,0 \mid 1,\lambda_4\rangle}_{1} \times \langle {}^3s_1 \\
&\quad + \sqrt{2/3}\langle 1,0,1,\lambda_4 \mid 1,\lambda_4\rangle \underbrace{\langle 1,\lambda_4,0,0 \mid 1,\lambda_4\rangle}_{1} \times \langle {}^3p_1 \\
&\quad + \sqrt{5/3}\langle 2,0,1,\lambda_4 \mid 1,\lambda_4\rangle \underbrace{\langle 1,\lambda_4,0,0 \mid 1,\lambda_4\rangle}_{1} \langle {}^3d_1] |T^1| \times \\
&\quad \times [\sqrt{3/3}\langle 1,0,0,\lambda_1-\lambda_2 \mid 1,\lambda_1-\lambda_2\rangle \langle 1/2,\lambda_1,1/2,-\lambda_2 \mid 0,\lambda_1-\lambda_2\rangle \,|^1 P_1\rangle + \\
&\quad + \sqrt{1/3}\langle 0,0,1,\lambda_1-\lambda_2 \mid 1,\lambda_1-\lambda_2\rangle \langle 1/2,\lambda_1,1/2,-\lambda_2 \mid 1,\lambda_1-\lambda_2\rangle \,|^3 S_1\rangle + \\
&\quad + \sqrt{3/3}\langle 1,0,1,\lambda_1-\lambda_2 \mid 1,\lambda_1-\lambda_2\rangle \langle 1/2,\lambda_1,1/2,-\lambda_2 \mid 1,\lambda_1-\lambda_2\rangle \,|^3 P_1\rangle + \\
&\quad + \sqrt{5/3}\langle 2,0,1,\lambda_1-\lambda_2 \mid 1,\lambda_1-\lambda_2\rangle \langle 1/2,\lambda_1,1/2,-\lambda_2 \mid 1,\lambda_1-\lambda_2\rangle \,|^3 D_1\rangle]
\end{aligned} \tag{64}$$

with $s, p, d, .. \hat{=} L_{34} = 0, 1, 2..;$ $\quad S, P, D, .. \hat{=} L_{12} = 0, 1, 2, ..$

Because of Parity-and C-Parity conservation eight of the twelfe amplitudes are zero:
Parity conservation:$P_{12} = (-1)^{L_{12}+1} = P_{34} = (-1)^{L_{34}}$
C-Parity conservation:$C_{12} = (-1)^{L_{12}+S_{12}} = C_{34} = -1$

The four remaining amplitudes are:

$$\begin{aligned}
&\langle \lambda_\mathbf{4}, \mathbf{0} \mid \mathbf{T^1} \mid \lambda_\mathbf{1}, \lambda_\mathbf{2} \rangle = \\
&\quad \sqrt{1/3}\langle 0,0,1,\lambda_4 \mid 1,\lambda_4\rangle \langle 1,\lambda_4,0,0 \mid 1,\lambda_4\rangle \sqrt{3/3}\langle 1,0,0,\lambda_1-\lambda_2 \mid 1,\lambda_1-\lambda_2\rangle \times \\
&\quad \times \langle 1/2,\lambda_1,1/2,-\lambda_2 \mid 0,\lambda_1-\lambda_2\rangle \langle \mathbf{{}^3s_1} \mid \mathbf{T^1} \,|\mathbf{{}^1\,P_1}\rangle + \\
&\quad + \sqrt{2/3}\langle 1,0,1,\lambda_4 \mid 1,\lambda_4\rangle \langle 1,\lambda_4,0,0 \mid 1,\lambda_4\rangle \sqrt{1/3}\langle 0,0,1,\lambda_1-\lambda_2 \mid 1,\lambda_1-\lambda_2\rangle \times \\
&\quad \times \langle 1/2,\lambda_1,1/2,-\lambda_2 \mid 1,\lambda_1-\lambda_2\rangle \langle \mathbf{{}^3p_1} \mid \mathbf{T^1} \,|\mathbf{{}^3\,S_1}\rangle + \\
&\quad + \sqrt{2/3}\langle 1,0,1,\lambda_4 \mid 1,\lambda_4\rangle \langle 1,\lambda_4,0,0 \mid 1,\lambda_4,\rangle \sqrt{5/3}\langle 2,0,1,\lambda_1-\lambda_2 \mid 1,\lambda_1-\lambda_2\rangle \times \\
&\quad \times \langle 1/2,\lambda_1,1/2,-\lambda_2 \mid 1,\lambda_1-\lambda_2\rangle \langle \mathbf{{}^3p_1} \mid \mathbf{T^1} \,|\mathbf{{}^3\,D_1}\rangle + \\
&\quad + \sqrt{5/3}\langle 2,0,1,\lambda_4 \mid 1,\lambda_4\rangle \langle 1,\lambda_4,0,0 \mid 1,\lambda_4\rangle \sqrt{3/3}\langle 1,0,0,\lambda_1-\lambda_2 \mid 1,\lambda_1-\lambda_2\rangle \times \\
&\quad \times \langle 1/2,\lambda_1,1/2,-\lambda_2 \mid 0,\lambda_1-\lambda_2\rangle \langle \mathbf{{}^3d_1} \mid \mathbf{T^1} \,|\mathbf{{}^1\,P_1}\rangle
\end{aligned} \tag{65}$$

B Integration over Φ_{34}

(19) can be written in the following form ($\lambda_5 = 0$):

$$\frac{d^4\sigma}{d\cos\Theta_{34} d\cos\Theta_{56} d\Phi_{34} d\Phi_{56}} = a_{iso}^2 \frac{3\,p_\gamma}{2048\,\pi^3\,p_{\overline{p}}^2\,m_\omega^2\,\Gamma_\omega} \times$$
$$\times \sum_{\lambda_1,\lambda_2,\lambda_6} \Big| \sum_J (2J+1) \sum_{\lambda_4} D^{J*}_{\lambda_1-\lambda_2,\lambda_4}(\Omega_{34})\, D^{1*}_{\lambda_4\lambda_5-\lambda_6}(\Omega_{56}) \times F^J_{\lambda_1\lambda_2\lambda_4\lambda_6} \Big|^2$$

with

$$F^J_{\lambda_1\lambda_2\lambda_4\lambda_6} = \sum_{L_{12},S_{12},L_{34},S_{34}(=1)} \sqrt{\frac{2L_{12}+1}{2J+1}} \sqrt{\frac{2L_{34}+1}{2J+1}} \langle L_{34}, 0, 1, \lambda_4 \mid J, \lambda_4 \rangle \times$$
$$\times \langle L_{12}, 0, S, \lambda_1-\lambda_2 \mid J, \lambda_1-\lambda_2 \rangle \times \langle 1/2, \lambda_1, 1/2, -\lambda_2 \mid S, \lambda_1-\lambda_2 \rangle \times \lambda_6 \times \hat{T}^{J^{PC}}_{L_{12},S_{12},L_{34},S_{34}(=1)}$$

$$\frac{d^4\sigma}{d\cos\Theta_{34} d\cos\Theta_{56} d\Phi_{34} d\Phi_{56}} = a_{iso}^2 \frac{3\,p_\gamma}{2048\,\pi^3\,p_{\overline{p}}^2\,m_\omega^2\,\Gamma_\omega} \times$$
$$\times \sum_{\lambda_1,\lambda_2,\lambda_6} \sum_{JJ\prime} (2J+1)(2J\prime+1) \sum_{\lambda_4\lambda_4\prime} [D^{J*}_{\lambda_1-\lambda_2,\lambda_4}(\Omega_{34}) D^{J\prime}_{\lambda_1-\lambda_2,\lambda_4\prime}(\Omega_{34})\, D^{1*}_{\lambda_4\lambda_5-\lambda_6}(\Omega_{56}) D^{1}_{\lambda_4\prime\lambda_5-\lambda_6}(\Omega_{56}) \times$$
$$\times F^J_{\lambda_1\lambda_2\lambda_4\lambda_6} F^{J\prime*}_{\lambda_1\lambda_2\lambda_4\prime\lambda_6}]$$

With

$$D^{J*}_{\lambda_1-\lambda_2,\lambda_4}(\Omega_{34}) D^{J\prime}_{\lambda_1-\lambda_2,\lambda_4\prime}(\Omega_{34})\, D^{1*}_{\lambda_4\lambda_5-\lambda_6}(\Omega_{56}) D^{1}_{\lambda_4\prime\lambda_5-\lambda_6}(\Omega_{56}) = \exp\big(\imath(\lambda_4-\lambda_4\prime)\underbrace{(\Phi_{56}-\Phi_{34})}_{\Phi_{56\prime}}\big) \times$$
$$\times d^J_{\lambda_1-\lambda_2\,\lambda_4}(\cos\Theta_{34}) d^{J\prime}_{\lambda_1-\lambda_2\,\lambda_4\prime}(\cos\Theta_{34}) d^1_{\lambda_4\lambda_5-\lambda_6}(\cos\Theta_{56}) d^1_{\lambda_4\prime\lambda_5-\lambda_6}(\cos\Theta_{56}) =$$
$$= d^J_{\lambda_1-\lambda_2\,\lambda_4}(\cos\Theta_{34}) d^{J\prime}_{\lambda_1-\lambda_2\,\lambda_4\prime}(\cos\Theta_{34}) \times D^{1*}_{\lambda_4\lambda_5-\lambda_6}(\Omega_{56\prime}) D^{1}_{\lambda_4\prime\lambda_5-\lambda_6}(\Omega_{56\prime})$$

the intergration over Φ_{34} results in

$$\frac{d^3\sigma}{d\cos\Theta_{34} d\cos\Theta_{56} d\Phi_{56}\prime} = \int \frac{d^4\sigma}{d\cos\Theta_{34} d\cos\Theta_{56} d\Phi_{34} d\Phi_{56}\prime} d\Phi_{34} =$$
$$= a_{iso}^2 \times \frac{3\,p_\gamma}{2048\,\pi^3\,p_{\overline{p}}^2\,m_\omega^2\,\Gamma_\omega} \sum_{\lambda_1,\lambda_2,\lambda_6} \sum_{JJ\prime} (2J+1)(2J\prime+1) \sum_{\lambda_4\lambda_4\prime} \int d^J_{\lambda_1-\lambda_2\,\lambda_4}(\cos\Theta_{34}) \times$$
$$\times d^{J\prime}_{\lambda_1-\lambda_2\,\lambda_4\prime}(\cos\Theta_{34}) d\Phi_{34} \times D^{1*}_{\lambda_4\lambda_5-\lambda_6}(\Omega_{56\prime}) D^{1}_{\lambda_4\prime\lambda_5-\lambda_6}(\Omega_{56\prime}) \times F^J_{\lambda_1\lambda_2\lambda_4\lambda_6} F^{J\prime*}_{\lambda_1\lambda_2\lambda_4\prime\lambda_6}]$$

$$= 2\pi \times a_{iso}^2 \times \frac{3\,p_\gamma}{2048\,\pi^3\,p_{\overline{p}}^2\,m_\omega^2\,\Gamma_\omega} \sum_{\lambda_1,\lambda_2,\lambda_6} \times$$
$$\times \Big| \sum_J (2J+1) \sum_{\lambda_4} d^J_{\lambda_1-\lambda_2\,\lambda_4}(\cos\Theta_{34}) \times D^{1*}_{\lambda_4\lambda_5-\lambda_6}(\Omega_{56\prime}) \times F^J_{\lambda_1\lambda_2\lambda_4\lambda_6} \Big|^2$$

C Final Amlitude for J=1 and J=2

J=1:

S_{12}=0 L_{12}=J=1(C-conservation) L_{34}=$0,\not{1},2$(P-conservation)

S_{12}=1 $L_{12}=0,\not{1},2$(C-conservation) $L_{34}=\not{0},1,\not{2}$(P-conservation)

$$\begin{aligned}
w = 2 \times \big|3 \sum_{\lambda_4} d^1_{0,\lambda_4}(\Theta_{34})\, D^{1*}_{\lambda_4\lambda_6-\lambda_5}(\Omega_{56'})\sqrt{1/2}[\sqrt{1/3}\langle 0,0,1,\lambda_4 \mid 1,\lambda_4\rangle \times T^{1^{+-}}_{1001} + \\
+ \sqrt{5/3}\langle 2,0,1,\lambda_4 \mid 1,\lambda_4\rangle \times T^{1^{+-}}_{1021}]\big|^2 + \\
+ 2 \times \big|3 \sum_{\lambda_4} d^1_{0,\lambda_4}(\Theta_{34})\, D^{1*}_{\lambda_4\lambda_6-\lambda_5}(\Omega_{56'})\sqrt{1/2}\langle 1,0,1,\lambda_4 \mid 1,\lambda_4\rangle[\sqrt{1/3} \times T^{1^{--}}_{0111} \; - \; \sqrt{2/3} \times T^{1^{--}}_{2111}]\big|^2 + \\
+ \big|3 \sum_{\lambda_4} d^1_{1,\lambda_4}(\Theta_{34})\, D^{1*}_{\lambda_4\lambda_6-\lambda_5}(\Omega_{56'})\sqrt{1/2}\langle 1,0,1,\lambda_4 \mid 1,\lambda_4\rangle[\sqrt{1/3} \times T^{1^{--}}_{0111} \; + \; \sqrt{1/6} \times T^{1^{--}}_{2111}]\big|^2 + \\
+ \big|3 \sum_{\lambda_4} d^1_{-1,\lambda_4}(\Theta_{34})\, D^{1*}_{\lambda_4\lambda_6-\lambda_5}(\Omega_{56'})\sqrt{1/2}\langle 1,0,1,\lambda_4 \mid 1,\lambda_4\rangle[\sqrt{1/3} \times T^{1^{--}}_{0111} \; + \; \sqrt{1/6} \times T^{1^{--}}_{2111}]\big|^2
\end{aligned} \quad (66)$$

with the four independent amplitudes:

$T^{1^{+-}}_{1001} = \langle {}^3s_1 \mid \hat{T} \mid^1 P_1\rangle$

$T^{1^{+-}}_{1021} = \langle {}^3d_1 \mid \hat{T} \mid^1 P_1\rangle$

$T^{1^{--}}_{0111} = \langle {}^3p_1 \mid \hat{T} \mid^3 S_1\rangle$

$T^{1^{--}}_{2111} = \langle {}^3p_1 \mid \hat{T} \mid^3 D_1\rangle$

J=2:

S_{12}=0 $L_{12}=J=\not{2}$(C-conservation)

S_{12}=1 $L_{12}=\not{1},2\;\not{3}$(C-conservation) $L_{34}=1,\not{2},3$(P-conservation)

$$\begin{aligned}
w = 2 \times \big|3 \sum_{\lambda_4} d^2_{0,\lambda_4}(\Theta_{34})\, D^{1*}_{\lambda_4\lambda_5-\lambda_6}(\Omega_{56'})\sqrt{1/2}[\sqrt{1/3}\langle 0,0,1,\lambda_4 \mid 1,\lambda_4\rangle \times 0\,\big|^2 \\
+ 2 \times \big|5 \sum_{\lambda_4} d^2_{0,\lambda_4}(\Theta_{34})\, D^{1*}_{\lambda_4\lambda_5-\lambda_6}(\Omega_{56'})\sqrt{5/3}\underbrace{\langle 2,0,1,0 \mid 2,0\rangle}_{0}\sqrt{1/2}[\sqrt{3/3}\langle 1,0,1,\lambda_4 \mid 2,\lambda_4\rangle \times T^{2^{--}}_{2111} \; + \\
+ \sqrt{7/3}\langle 3,0,1,\lambda_4 \mid 2,\lambda_4\rangle \times T^{2^{--}}_{2131}]\big|^2 + \\
+ \big|5 \sum_{\lambda_4} d^2_{1,\lambda_4}(\Theta_{34})\, D^{1*}_{\lambda_4\lambda_5-\lambda_6}(\Omega_{56'})\sqrt{5/3}\underbrace{\langle 2,0,1,1 \mid 2,1\rangle}_{-\sqrt{1/2}}[\sqrt{3/3}\langle 1,0,1,\lambda_4 \mid 2,\lambda_4\rangle \times T^{2^{--}}_{2111} \; + \\
+ \;,\sqrt{7/3}\langle 3,0,1,\lambda_4 \mid 2,\lambda_4\rangle \times T^{2^{--}}_{2131}]\big|^2 + \\
+ \big|5 \sum_{\lambda_4} d^2_{-1,\lambda_4}(\Theta_{34})\, D^{1*}_{\lambda_4\lambda_5-\lambda_6}(\Omega_{56'})\sqrt{5/3}\underbrace{\langle 2,0,1,-1 \mid 2,-1\rangle}_{-\sqrt{1/2}}[\sqrt{3/3}\langle 1,0,1,\lambda_4 \mid 2,\lambda_4\rangle \times T^{2^{--}}_{2111} \; + \\
+ \sqrt{7/3}\langle 3,0,1,\lambda_4 \mid 2,\lambda_4\rangle \times T^{2^{--}}_{2131}]\big|^2
\end{aligned} \quad (67)$$

with the two independent amplitudes(only M=$\pm$ 1-terms)
$T^{2--}_{2111} = \langle ^3p_2 \mid \hat{T} \mid 3D_2 \rangle$
$T^{2--}_{2131} = \langle ^3f_2 \mid \hat{T} \mid 3D_2 \rangle$

Note:The M=1 amplitude contributes in $\sum_J$ with a negative sign,the M=-1 amplitude with a positive sign.